Physics of the Earth and Planetary Interiors 111 (1999) 21–34

PHYSICS OF THE EARTH AND PLANETARY INTERIORS

Dynamics of the stably stratified ocean at the top of the core

Stanislav I. Braginsky *

Institute of Geophysics and Planetary Physics, University of California Los Angeles, Los Angeles CA 90095 1567, USA

Received 8 December 1997; accepted 30 April 1998

Abstract

It is supposed that the stably stratified layer exists at the top of the fluid core. We propose a new model of this layer, which we call the Stratified Ocean of the Core (SOC). The SOC has a density that differs very little from the one corresponding to the adiabatic density gradient, but this small difference implies a very large buoyancy force. Therefore, hydrodynamic properties of such a thin layer differ drastically from those of the bulk of the Earth's core. There is a close similarity between the SOC and the Earth's common ocean: both of them are a thin shell, and their Brunt–Väisälä frequencies are of the same order of magnitude. New features are added to the SOC due to the effects of the core magnetic field. We consider various kinds of waves, with periods ranging from days to decades, which can develop in the SOC. The oscillations in the SOC with periods about decades determine the decade geomagnetic and length-of-day variations. The topographic core–mantle coupling, arising due to perturbation of fluid motion in the SOC by the core–mantle boundary unevenness, is considered. The key problem of proving the very existence of the SOC is discussed. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Hydrodynamic properties; Earth; Core–mantle coupling

1. Introduction

An assumption that the top of the core, adjacent to the core–mantle boundary (CMB), is stably stratified has been discussed for a long time (see Whaler, 1980; Fearn and Loper, 1981; Yukutake, 1981; Gubbins et al., 1982; Frank, 1982; Braginsky, 1984, 1987a; Bergman, 1993; Braginsky, 1993; Braginsky and Le Mouël, 1993; Lister and Buffett, 1994; Loper and Lay, 1995; Braginsky, 1998; Shearer and Roberts, 1997). The convincing proof of the existence of a stably stratified layer at the top of the core is still absent, however, and the structure of the core's top is to be investigated.

Braginsky (1993) has shown that two observed effects can be explained by the same physical process in this layer, namely that the 65-year variations of both the geomagnetic field and the speed of the Earth's rotation can be generated by an axially symmetric oscillation in the layer, akin to MAC-waves. This explanation of the mechanism of 65-year oscillations provides an observational support to the assumption of existence of the stably stratified layer at the top of the core. The stably stratified layer is characterized by its density excess, $C = (\rho - \rho_a)/\rho_a$, which is negative. Here ρ is the fluid true density, ρ_a is the equilibrium density corresponding to the adiabatic gradient; we substitute ρ_a below by the

* Corresponding author. E-mail: sbragins@igpp.ucla.edu

PII: S0031-9201(98)00143-5

constant $\rho_0 = 10$ g cm^{-3}. Braginsky (1993) assumed the following three-parameter model of the stratified layer with the linear $C(r)$ dependence

$$C = -C_S - (C_H/H)(r - R_S). \quad (1)$$

The CMB placed at the radius R_1 is the bottom of the layer, and its top merges with the bulk of the core at the radius $R_S = R_1 - H$, where the layers' density excess, C, drops from C_S to a negligibly small value, $\sim C_0$, corresponding to the bulk of the core. The density excess, $\sim C_0$, driving the geodynamo in the bulk of the core, is extremely small ($C_0 \sim 10^{-8}$ or smaller) and we ignore it here. It is convenient to use two corresponding Brunt–Väisälä frequencies instead of C_H and C_S:

$$N = (g_1 C_H/H)^{1/2}, \quad (2a)$$

$$N_S = (g_1 C_S/H)^{1/2}, \quad (2b)$$

where g is the gravity acceleration; $g_r = -g_1$ is assumed constant in the SOC, $g_1 = 10$ m s^{-2}.

Two parameters of the layer were estimated in Braginsky (1993) by comparing the theory of the 65-year oscillation, with the observed variations of the geomagnetic dipole and of the length of day. The estimated parameters of the layer are its thickness, $H \approx 80$ km, and the Brunt–Väisälä frequency, $N \approx 2\Omega$. Here, $\Omega = 0.729 \times 10^{-4}$ s^{-1} is the angular velocity of Earth's daily rotation. These parameters correspond to $C_H \sim 10^{-4}$. Even the best available seismic models, e.g., the well known PREM model, are not able to distinguish such a small deviation of the core density distribution from the adiabatic stratification. That is why the stratified layer was called in Braginsky (1993) 'the *hidden* ocean of the core', or HOC. The evidence is now accumulated from seismic observations suggesting $\sim 1\%$ decline in the seismic velocity in the uppermost layer of the core about 50–100 km thick (see, e.g., Lay and Young, 1990; Souriau and Poupinet, 1991; Garnero et al., 1993; and Sylvander and Souriau, 1996). This effect is not reliably measured yet, because of difficulties associated with the D$''$ layer inhomogeneity in the mantle nearby, but if confirmed, it would provide a direct proof of the existence of the layer of light material at the top of the core. This material floats on the underlying bulk of the core being pressed to the CMB by the Archimedean buoyancy force, hence it is the 'light material'. It has the deficit of $\sim 1\%$ in seismic velocity, and having no additional information we assume that it has the density deficit of the same order of magnitude, therefore $C_S \sim 10^{-2}$. The density jump, $C_S \sim 10^{-2}$, is two orders of magnitude greater than C_H, and corresponds to the Brunt–Väisälä frequency, $N_S = (gC_S/H)^{1/2}$, which is one order of magnitude greater than the value $N \sim 2\Omega$ estimated in Braginsky (1993). This apparent contradiction can be resolved in the frame of the model (1) by recognizing that the Brunt–Väisälä frequency, N, found from the MAC-oscillation in the layer is determined by the value of inhomogeneity inside the layer, C_H, while seismic measurements are sensitive to the much greater total magnitude of C, which is approximately equal to the jump, C_S. This jump separates the layer from the bulk of the core, like the density jump on the surface of the common ocean separates it from the atmosphere. In anticipation of a future confirmation of this picture of the layer of light material at the top of the core we remove the adjective 'hidden' from its name and call the layer simply the 'Stratified Ocean of the Core' (SOC). The SOC can be observed through its dynamic effects which are rather strong because the Brunt–Väisälä frequency in the layer, N, is about three orders of magnitude greater than the one in the bulk of the core, which is of order of $N_0 \sim (gC_0/R_1)^{1/2}$. The jump, C_S, is about six orders of magnitude greater than the density excess, C_0, in the bulk of the core, and this makes the surface of the layer, $r = R_S$, very rigid. It should be emphasized that even $C \approx C_S \sim 1\%$ is still much smaller than unity.

There is a close similarity between the SOC and the Earth's common ocean, both in their geometry of a thin shell and in the magnitude of the Brunt–Väisälä frequency. The dynamics of the SOC is reminiscent of the rich dynamics of the ocean and atmosphere, which are known to be brought about by the joint action of stratification and Coriolis force through manifold processes such as surface waves, gyroscopic (inertial) waves, internal gravity waves, Rossby waves, baroclinic instability, etc. Similar processes can be expected to develop in the SOC but they are seriously modified by magnetic field of the core. Additional kind of motion—MAC oscillations, unknown in oceanology, is possible in the SOC due to the core's magnetic field.

The core–mantle coupling which originates from the interaction of the surface flow with the topography of the core–mantle boundary, is strongly influenced by the stably stratified layer. We consider the topographic core–mantle coupling arising due to generation of motion resembling the magnetic Rossby waves in the stably stratified layer. A simple expression is obtained for the topographic tangential stress on the core–mantle boundary (Braginsky, 1998).

The SOC is created by the arrival of light fluid into the layer from the bulk of the core and (maybe) also from the mantle, and by unknown processes of the removal of light fluid from the layer, as well as by light fluid mixing (turbulent diffusion) in the layer. It is tempting to call the processes of addition of light fluid from the bulk of the core and of its removal 'rain' and 'evaporation'. The leakage of light material from the mantle to the fluid core due to chemical reactions on the CMB was suggested by Jeanloz (1990). An anonymous referee of (Braginsky, 1993) named this source 'artesian wells'. Physical processes behind all these names are poorly known. It was not even tried here to investigate the obscure mechanisms of the SOC formation. Instead the model (1) is postulated, and various types of waves which can develop in such an ocean of conducting fluid penetrated by magnetic field are considered. This provides us with an insight into the physics of the SOC, and with an opportunity to bring together the SOC theory and geophysical observations. After the existence of the SOC is proven and its parameters are known, one can better speculate about the mechanism of the SOC formation.

Many problems could be associated with a rich dynamics of the SOC. For example, a mechanism of the geodynamo could depend significantly on the structure of the SOC and on the process of its formation. The decade variations of geomagnetic field and variations of the Earth's rotation are connected with the processes in the SOC. Various unusually fast processes in the core ('jerks', etc.) also could be associated with the SOC.

The present paper is mainly a review of the author's previous original works. It summarizes the analyses of Braginsky (1993) in Section 3, and Braginsky (1998) in Sections 4 and 5; the fast processes in the SOC, with frequencies $\sim \Omega$, are discussed in Section 6; Section 2 describes the equations governing dynamics of the SOC. To prove the very existence of the SOC one have to develop the SOC theory and compare it with observational data. This key problem, which is a subject of the author's current work, is discussed in Section 7.

2. Main equations

2.1. The equations

The SOC dynamics can be described by the well known system of hydromagnetic equations for the fluid velocity, V, the magnetic field, B, and the density excess, C, in the Boussinesq approximation. We are working mostly with the linear oscillations; therefore, it is convenient to write $V+v$, $B+b$, and $C+c$ instead of V, B, and C. Here the capital and the small letters denote the basic and the oscillating quantities, respectively. The linearized hydromagnetic equations in the Boussinesq approximation may be written as

$$d_t \boldsymbol{v} + 2\boldsymbol{\Omega}\times\boldsymbol{v} = -\nabla p + \boldsymbol{g}c + (\boldsymbol{B}\cdot\nabla)\boldsymbol{b} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{B}, \tag{3}$$

$$\partial_t \boldsymbol{b} - \eta\nabla^2\boldsymbol{b} = \nabla(\boldsymbol{v}\times\boldsymbol{B} + \boldsymbol{V}\times\boldsymbol{b}), \tag{4}$$

$$d_t c = -\boldsymbol{v}\cdot\nabla C, \tag{5}$$

$$\nabla\cdot\boldsymbol{v} = 0, \tag{6a}$$

$$\nabla\cdot\boldsymbol{b} = 0. \tag{6b}$$

Here, $d_t = \partial_t + \boldsymbol{V}\cdot\nabla$ is a linearized approximation to the material time derivative. The constant values $\rho_0 \approx 10$ g/cm^3 and $g \approx 10^3$ cm/s^2 are adopted for the SOC. The magnetic field is divided by $(\mu_0\rho_0)^{1/2}$, and therefore has the dimension of velocity, 1 cm/s corresponding to 11.2 G. The 'effective' pressure, $p = p_T + \boldsymbol{B}\cdot\boldsymbol{b}$, is divided by ρ_0; here p_T is the hydrostatic pressure.

2.2. The boundary conditions

The boundary conditions on the surface of the ocean $(r = R_S)$ and on its bottom $(r = R_1)$ are determined by properties of the bulk of the core and of the mantle. The normal (radial) velocity component and all components of the magnetic field should be continuous:

$$[\![v_r]\!] = 0 \text{ at } r = R_S, \tag{7}$$

$$[\![\boldsymbol{b}]\!] = 0 \text{ at } r = R_S, \qquad r = R_1. \tag{8}$$

Here, double square brackets denote the jump in quantity. Jumps in tangential velocities, v_τ, are admissible because we neglect viscosity and disregard the Ekman layers, which are very thin. On the solid CMB, at $r = R_1$, we have $v_r = 0$, and an arbitrary pressure is admissible there. The pressure perturbation can have a jump at $r = R_S$, which is subject to the condition of equilibrium (see Braginsky, 1993):

$$[\![\partial_t p]\!] = -HN_S^2 v_r(R_S). \tag{9}$$

For fast processes, with frequencies $\sim \Omega$, and large inertia forces, this condition determines tangential velocity jump at the SOC surface, $r = R_S$. For slow processes, with small inertia forces, it is approximately equivalent to a simpler condition valid for a solid surface, $v_r(R_S) = 0$, because N_S^2 is very large. In the oceanology this simplification is called 'a rigid lid approximation'.

2.3. The energy balance equation

The energy balance equation can be obtained as a sum of Eq. (3) multiplied by $\boldsymbol{v}\cdot$, Eq. (4) multiplied by $\boldsymbol{b}\cdot$, and Eq. (5) multiplied by the combination $cg_r/\partial_r C$. This gives after a simple transformation

$$\partial_t \varepsilon_\Sigma + \nabla \times \boldsymbol{i}^\varepsilon = -\eta j^2, \tag{10}$$

where $\varepsilon_\Sigma = \varepsilon_v + \varepsilon_b + \varepsilon_\alpha$, $\varepsilon_v = v^2/2$, $\varepsilon_b = b^2/2$, $\varepsilon_\alpha = (g_r^2/N^2)c^2/2$, and $\boldsymbol{i}^\varepsilon = p_T\boldsymbol{v} + \boldsymbol{e} \times \boldsymbol{b}$, $\boldsymbol{e} = \eta \mathbf{j} - \boldsymbol{v} \times \boldsymbol{B}$, $\boldsymbol{j} = \nabla \times \boldsymbol{b}$. The flux of energy, $\boldsymbol{i}^\varepsilon$, consists of the mechanical flux due to the oscillating hydrostatic pressure, p_T, and the electromagnetic flux (Pointing's vector), $e \times b$, where the electric field, e, is determined by Ohm's law. Both fluxes are expressed in specific units proportional to $(\mu_0 \rho_0)^{1/2}$. On the right hand side of the energy equation stands the Joule heat production per unit mass. All energy densities and the squared magnetic field are divided for convenience by the (constant) fluid density, ρ_0. Integrating Eq. (10) over the space, we obtain the integral energy balance

$$\partial_t E_\Sigma = -Q_j, \tag{11}$$

where

$$E_\Sigma = \int \varepsilon_v \mathrm{d}V + \int \varepsilon_b \mathrm{d}V + \int \varepsilon_\alpha \mathrm{d}V + \int \varepsilon_S \mathrm{d}A,$$

and $\varepsilon_S = N_S^2 h_S^2/2$.

Here, ε_v is integrated over the core's volume, ε_α is integrated over the volume of the SOC, ε_S is integrated over the surface of the SOC, and ε_b is integrated over the whole space, hence the electromagnetic energy flux, $\boldsymbol{e} \times \boldsymbol{b}$, disappears because the external energy input is absent. The total energy dissipation is mostly produced by the Joule heating $Q_J = \int \eta\ j^2 \mathrm{d}V$. It determines the decay of oscillations. The dissipation due to viscous friction, Q_ν, depends on the value of viscosity which is poorly known. The commonly accepted value, $\nu \sim 10^{-6}$ m^2 s^{-1} $\sim 10^{-6}\eta$, corresponds to the Ekman number as small as $\sim 10^{-15}$. We will ignore Q_ν.

3. Axisymmetric MAC-oscillations

Decade variations of both geomagnetic field and length of day (l.o.d.) results from the mechanism of the decade oscillations of the SOC. The l.o.d. variation is more convenient for the diagnostic of these oscillations because it is known with much better precision and over a longer time interval than the geomagnetic variations. A simple analytical expression, which gives a very good approximation of decade l.o.d. variation, was obtained (Braginsky, 1984) by fitting the observational data of Morrison (1979) for the interval 1861–1978. The fitting is based on the assumption that the variation is composed of a decaying mode of the frequency $\omega_1 = 2\pi/\tau_1$, and a nearly stationary oscillation of the frequency $\omega_2 = 2\omega_1$ (with addition of a resonance mode $\omega_3 = \omega_1 + \omega_2$). This least square fit gave $\tau_1 = 65$ years, and provided the values of amplitudes and phases of all modes. The reliability of this result is highly increased by the fact that the l.o.d. variation observed for more than ten years after 1978 is well *predicted* by the fitted analytical expression. The inspection of the l.o.d. prior to 1861 shows (Braginsky, 1987b) that the decaying ω_1 mode started shortly before this year while the ω_2 oscillation persisted for a long time.

What is the dynamic mechanism of the decade variations? It is shown by Braginsky (1993) that the axisymmetric MAC-oscillations in the SOC similar to MAC-waves are possible in the SOC. A qualitative picture of these SOC oscillations (SO for brevity) is determined by mutual equilibration of Magnetic, $\boldsymbol{f}^B$, Archimedean, $\boldsymbol{f}^\alpha$, and Coriolis, $\boldsymbol{f}^\Omega$, forces, together with the pressure gradient, as is common for the MAC-waves. The following order of magnitude estimates can be written for these forces in the MAC-oscillations of the SOC:

$$f_r^\alpha \sim N^2 u_r,\ \partial_r p \sim k_H p,\ \left(f_\theta^B, f_\phi^B\right) \sim k_H^2 B_r^2 (u_\theta, u_\phi),$$
$$\left(f_\theta^\Omega, f_\phi^\Omega\right) \sim 2\Omega (v_\phi, v_\theta), \tag{12}$$

where $k_H = \pi/H$, $\nabla_\theta p \sim p/L$. Here $\boldsymbol{v} = -i\omega\mathbf{u}$ is the fluid velocity, and u is the fluid particles' displacement in the oscillations. A qualitative relations of equilibrium and continuity:

$$f_r^\alpha \sim \partial_r p,\ f_\theta^\Omega \sim \nabla_\theta p,\ f_\theta^B \sim f_\phi^\Omega,$$
$$\text{and } u_\theta/u_r \sim v_\theta/v_r \sim k_H L, \tag{13}$$

with the above estimate of the forces give the frequency of free SO oscillations, $\omega_{SO} \sim \omega_A N/2\Omega$. Here $\omega_A \sim B_r L^{-1}$ is the Alfvén frequency, and $v_\theta / v_\phi \sim k_H^2 B_r^2 / 2\Omega\omega$. The velocity v_ϕ in the SO is one order of magnitude greater than v_θ.

A more accurate picture of the SO was obtained by Braginsky (1993) through the solution of the system (3)–(6) for the spherical model of the SOC, with the artificial dipole magnetic field, $B_{rd} = B_d \cos\theta$, assumed for mathematical simplicity, instead of the real radial field, B_r. Here the constant $B_d = \langle B_r^2/(\cos\theta)^2\rangle^{1/2}$ represents the 'equivalent dipole'; the value $B_d = 5$ G (0.45 cm/s) was accepted. This assumption has no firm basis but makes it possible to separate variables r and θ in the equation of oscillations, thus avoiding numerical calculations. A simple approximate solution expressed through the associated Legendre functions $P_n^1(\cos\theta)$ was found, with the eigenfrequencies, $\omega_{SO}(n)$, proportional to $[n(n+1)]^{1/2}$:

$$\omega_{SO} \approx [n(n+1)]^{1/2} (B_d/R_1)(N/2\Omega). \tag{14}$$

Assuming that the mode $\omega_1 = 2\pi/\tau_1 = 3.07 \times 10^{-9}$ s^{-1} (here $\tau_1 = 65$ years) corresponds to the solution with $n = 2$, we have $\omega_{SO} \approx 3.17 \times 10^{-9}$ s^{-1} $(N/2\Omega)$. The condition $\omega_{SO} = \omega_1$ implies $N/2\Omega \approx 1$. A more rough solution for the next mode corresponds to $n = 4$.

The l.o.d. oscillation is produced by dragging the mantle with a rather big oscillating velocity v_ϕ. The amplitudes of the l.o.d. oscillation, Λ_d, and of the axial geomagnetic dipole oscillation, $\tilde{g}_0^1$, (observed for $\tau_1 = 65$ years) are proportional to the amplitude of the 65-year mode. The formula

$$H = 80\left(k_u \Lambda_d / \tilde{g}_0^1\right)^{1/3} \tag{15}$$

was derived by Braginsky (1993) using the angular momentum balance and the SO theory. Here $k_u \sim 1$ is a 'coefficient of uncertainty', H is expressed in km, Λ_d in ms, and $\tilde{g}_0^1$ in mG. It was obtained in Braginsky (1984) from observational data that $\Lambda_d \approx 1$ ms and $\tilde{g}_0^1 \approx 1$ mG, hence $H \approx 80$ km. The power $1/3$ in Eq. (15) reduces the influence of all uncertainties. Note that $H \approx 80$ km is significantly greater than the skin-effect length, $\delta_\eta = (2\eta/\omega)^{1/2}$, which is 36 km for the period $\tau_1 = 65$ years, and 26 km for $\tau_2 = 33$ years; here $\eta = 2$ m^2 s^{-1} is accepted.

Global torsional oscillations (TO), which are possible in the bulk of the core, are essentially the standing Alfvén waves (Braginsky, 1970, 1984). Their frequency, ω_{TO}, is of the order of Alfvén waves frequency, ω_A, corresponding to the properly averaged field B_S. It can be seen from Eq. (14) that for $N \sim 2\Omega$ we have $\omega_{TO} \sim \omega_{SO}$. The joint action of SO and TO provides the mechanism of generation of the decade geomagnetic and l.o.d. variations. The excitation mechanism of these oscillations is not clear. It is shown in (Braginsky, 1993) that TO can be excited by SO, and perhaps SO can be excited by the 'sloping instability' (if C_H depends on θ) and/or by some unknown perturbation. The steadiness of the ≈ 30 years oscillation found in the l.o.d. variation poses an especially complicated problem of their permanent support. The period ≈ 30 years was found in variations of the geomagnetic Gauss coefficients (Yokoyama, 1993), and it is revealed also in the variation of the Earth's rotation pole position (see, e.g., Lambeck, 1980; Hulot et al., 1996). Torsional oscillations in the bulk of the core also have free periods ~ 30 years. Section 4 deals with the non-axisymmetric 'magnetic Rossby' (MR) waves in the SOC. The period of about 30 years may be expected for these waves of global-scale, with $m=1$, 2. Further investigations are necessary to understand the complicated mechanism of ≈ 30-year variations, which may include the non-linear interaction between various kinds of axisymmetric and non-axisymmetric oscillations in the core. Results of such investigations will be helpful in understanding the nature of secular variations, as well as in improving our knowledge of the SOC parameters, and of the core–mantle interaction.

4. Magnetic Rossby waves

The SOC oscillations (SO) of Section 3 have no analog in the common ocean. The SO are mainly axisymmetric, $\nabla_\phi p=0$, and an essential part of their mechanism is the magnetic force which equilibrates the ϕ-component of Coriolis force. A different class of motions in the SOC consists of oscillations with ϕ-dependence, which we take to be proportional to $\exp(\mathrm{i}m\phi)$. This class of oscillations considered in Braginsky (1998), and with somewhat different model of the SOC in Braginsky (1984, 1987a), is similar to the Rossby waves, well-known in the oceanology (see, e.g., Brekhovskikh and Goncharov, 1985; Cushman-Roisin, 1994). The Rossby waves are often treated as a local perturbation, using the infinite plane model of the spherical ocean, and we follow this practice. We model the SOC by a plane fluid layer of thickness H placed between two horizontal solid walls, corresponding to $r=R_1$ and $r=R_S=R_1-H$. Let us assume that the horizontal scale of perturbations, L_τ, satisfies both locality of perturbation and a thin layer conditions: $H \ll L_\tau \ll R_1$. Let the local Cartesian coordinates be $[x_r=r-R_1$, $x_\theta=R_1(\theta-\theta_0)$, $x_\phi=s_0\phi]$, where $s_0=R_1 \sin\theta_0$. Directions (θ, ϕ) tangential to the spherical CMB at the colatitude θ_0 are symbolized by the subscript τ. For example, a horizontal gradient operator is $\nabla_\tau=\mathbf{1}_\theta\nabla_\theta+\mathbf{1}_\phi\nabla_\phi$, where $\nabla_\theta=R_1^{-1}\partial_\theta$ and $\nabla_\phi=s_0^{-1}\partial_\phi$. The magnetic field, B_r, is considered constant across the SOC. Both B_r and the 'Coriolis parameter', $2\Omega_r=2\Omega\cos\theta_0$, are taken at $\theta=\theta_0$. The change of the Coriolis parameter along the colatitude is taken into account in the linear approximation. It is measured by the constant $\beta=-\nabla_\theta 2\Omega_r=2\Omega R_1^{-1}\sin\theta_0$. This model corresponds to the well-known 'β-plane approximation', which is customary in oceanology and meteorology. We assume $\theta_0=45°$ and $\beta=3\times 10^{-11}\ \mathrm{m}^{-1}\ \mathrm{s}^{-1}$ for estimates.

The equations of the model are greatly simplified by the fact that Coriolis and Archimedean forces are strongly dominating. The inertia force is much smaller than the Coriolis force because $d_\mathrm{t} \ll \Omega$. The dimensionless number $B_\mathrm{r}^2/2\Omega_r\eta$ compares magnetic and Coriolis forces; it is of order of 10^{-1} or smaller. A quasistatic equilibrium of the two greatest vertical forces is established, $f_r^\alpha+\partial_r p=0$. The horizontal gradient of pressure is $\sim L_\tau/H \sim 10$ times smaller than the vertical one. It is balanced (approximately) by the horizontal component of the Coriolis force, which is about $2\Omega_r\eta/B_\mathrm{r}^2 \sim 10$ times greater than the magnetic force. Thus, the equation of motion is reduced in the leading approximation to the equation of horizontal equilibrium of the Coriolis force and the pressure gradient. It is called a 'tangentially geostrophic balance' equation.

The vertical velocity is much less than the horizontal one, and estimates show that even $v_r/v_\tau \ll H/L_\tau \ll 1$. Hence in the leading approximation we have $\nabla_\tau \times \boldsymbol{v}_\tau=0$. The quantity $\partial_r v_r$ does not enter the continuity equation in the leading approximation. It should be found from the higher approximation of the equation of motion. Fortunately, it is not necessary to construct a complicated perturbation scheme to derive the equation for v_r. Taking the *curl* of the equation of motion and looking on its vertical projection we eliminate both biggest terms, $\boldsymbol{f}^p=-\nabla p$ and $\boldsymbol{f}^\alpha=-\mathbf{1}_r g_1 c$, and obtain the equation governing the small deviations from the geostrophy, which include the small term $\partial_r v_r$. With the above simplifications, Eqs. (3)–(5), (6a) and (6b) can be reduced to the 'quasigeostrophic' system, well-known in the oceanology (but with the magnetic field, b, added)

$$g_1 c+\partial_r p=0, \tag{16}$$

$$2\Omega_r \mathbf{1}_r \times \boldsymbol{v}_\tau+\nabla_\tau p=0, \tag{17}$$

$$-2\Omega_r\partial_r v_r-\beta v_\theta=B_\mathrm{r}\partial_r(\nabla_\tau \times \boldsymbol{b}_\tau)_r, \tag{18}$$

$$d_\mathrm{t}\boldsymbol{b}-\eta\partial_r^2\boldsymbol{b}=B_\mathrm{r}\partial_r\boldsymbol{v}, \tag{19}$$

$$g_1 d_\mathrm{t} c-N^2 v_r=0, \tag{20}$$

$$\nabla_\tau \cdot \boldsymbol{v}_\tau=0, \tag{21a}$$

$$\nabla_\tau \cdot \boldsymbol{b}_\tau=0. \tag{21b}$$

The 'inertia term', $d_r(\nabla_\tau \times \boldsymbol{v}_\tau)_r$, which would appear in the left hand side of Eq. (18), is much smaller than βv_θ, and it is omitted. With the assumptions $(k_\theta R_1)^{-1} \sim 10^{-1}$ and $H/L_\theta \sim 10^{-1}$ the overall accuracy of quasigeostrophic theory of the SOC is about 10%. It is of course rougher than corresponding approximations in oceanology applications, because $H \sim 80$ km is much greater than a typical depth of the common ocean, and $R_1 = 3.48 \times 10^3$ km is smaller than the Earth's core radius $R_0 = 6.37 \times 10^3$ km. We hope, nevertheless, that the results obtained are at least qualitatively correct.

The systems (16)–(21) allows for a separation of variables. The solution can be sought in the form of a progressive wave in ϕ, but a standing wave in θ. In these waves (v_ϕ, b_ϕ) are proportional to $\sin(k_\theta x_\theta)\exp(i\Phi)$, and all other components are proportional to $\cos(k_\theta x_\theta)\exp(i\Phi)$. Here $\Phi = ik_\phi x_\phi - i\omega t$, $k_\theta = \pi/L_\theta$, $k_\phi = m/s_0 = \pi/L_\phi$, hence $k_\phi x_\phi = m\phi$, and $k_\tau^2 = k_\theta^2 + k_\phi^2$, where L_θ, L_ϕ, $L_\tau = \pi/k_\tau$ are the characteristic lengths. After the separation of variables is done, we substitute $\nabla_\phi = ik_\phi$, $\nabla_\theta^{\,2} = -k_\theta^2$, and replace d_t by $-i\overline{\omega}$, where $\overline{\omega} = \omega - k_\phi V_\phi$. Equations for b_θ, v_θ, and v_r can be derived from Eqs. (16)–(20), (21a) and (21b):

$$i\overline{\omega}(2\Omega_r/N)^2 \partial_r^2 v_\theta - \beta i k_\phi v_\phi = k_\tau^2 B_r \partial_r b_\theta, \tag{22}$$

$$i\overline{\omega} b_\theta + \eta \partial_r^2 b_\theta = -B_r \partial_r v_\theta. \tag{23}$$

$$v_r = \lambda_{r\theta} \partial_r v_\theta, \tag{24a}$$

$$\lambda_{r\theta} = -(\overline{\omega}/k_\phi)(2\Omega_r/N^2). \tag{24b}$$

The length $\lambda_{r\theta}$ is very small, e.g., for $2\pi/\overline{\omega} \sim 20$ years and $k_\phi \sim 4 \times 10^{-6}$ m^{-1} we have $\lambda_{r\theta} \sim 25$ m.

The bottom surface of the SOC is solid; its top surface, $r = R_S$, though fluid, is very rigid (effectively solid) because C_S is very large. Therefore, the boundary conditions for velocities are

$$v_r(R_1) = 0, \tag{25a}$$

$$v_r(R_S) = 0 \tag{25b}$$

or

$$\partial_r v_\theta(R_1) = 0, \tag{26a}$$

$$\partial_r v_\theta(R_S) = 0. \tag{26b}$$

A potential magnetic perturbation in the mantle, $\boldsymbol{b}^M = -\nabla\Psi$, changes in all directions on the same characteristic length, $L \sim 10^3$ km, thus throughout the mantle we have $b_\tau^M \sim b_r^M$. The same is true at the CMB, hence b_τ on the CMB is much smaller than inside the SOC, where $b_\tau \gg b_r$. We may, therefore, take, as a good approximation, a simple boundary condition, $b_\tau(R_1) = 0$. Magnetic and velocity perturbations in the bulk of the core, $\boldsymbol{b}^L$ and $\boldsymbol{v}^L$, which are generated by perturbations in the SOC, have a complicated form of superposition of MAC-waves. There is no specific reason for these waves to have $b_r^L \ll b_\tau^L$, and one may anticipate $b_\tau^L \sim b_r^L$. If this is correct then $b_\tau(R_S) = b_\tau^L \sim b_r^L = b_r(R_S) \ll b_\tau$ (inside the SOC), and the simplification $b_\tau = 0$ for $r = R_S$ can be assumed the same way as for $r = R_1$. This simplification gives approximately correct eigenfunctions and eigenfrequencies, but it eliminates magnetic stresses at $r = R_S$, therefore the energy transfer to the underlying bulk region of the core is omitted. It is shown, however, that the considered waves have a significant mechanism of decay due to the Joule heating inside the layer. In order to work with the simplest model we accept

$$b_\tau(R_1) = 0, \tag{27a}$$

$$b_\tau(R_S) = 0. \tag{27b}$$

Eqs. (22) and (23) with the boundary conditions Eqs. (26a), (26b), (27a) and (27b) have solutions

$$(v_\theta, v_\phi) = v_a(1, -ik_\theta/k_\phi)\cos\alpha_n, \tag{28a}$$

$$(b_\theta, b_\phi) = b_a(1, -ik_\theta/k_\phi)\sin\alpha_n, \tag{28b}$$

Here, $\alpha_n = nk_H(r - R_1)$, where $k_H = \pi/H$ and n is an integer. Eq. (23) gives the amplitude ratio, $b_a/v_a = (k_r B_r)(i\overline{\omega} - \eta k_r^2)^{-1}$. Eqs. (28a) and (28b) satisfy the continuity Eqs. (21a) and (21b). The radial velocity $v_r = v_{ra}\sin\alpha_n$, and the density perturbation $c = ic_a \sin\alpha_n$, where $v_{ra}/v_a = -\lambda_{r\theta} nk_H$, and $g_1 c_a/v_a = 2\Omega_r/k_\phi nk_H$, are given by Eqs. (20) and (24a). The eigenfrequencies $\overline{\omega}$ are to be found from the equation

$$(\overline{\omega} + i\eta k_r^2)(\omega_\beta k_H^2 + \overline{\omega} k_r^2) - \omega_B^2 k_r^2 = 0, \tag{29}$$

where the following quantities, ω_β and ω_B, with dimension of frequency are introduced:

$$\omega_\beta = \beta k_\phi k_H^{-2} (N/2\Omega_r)^2, \tag{30a}$$

$$\omega_B^2 = k_\tau^2 B_r^2 (N/2\Omega_r)^2 \tag{30b}$$

Let us assume $L_\theta = 10H = 8\times10^2$ km, $k_\theta = 3.9\times10^{-6}$ m^{-1}, and $L_\phi = \pi s_0/m = 1.55\times10^3(5/m)$ km, where $s_0 = 0.707R_1$; we take $m=5$, $k_\phi = 2\times10^{-6}$ m^{-1}, $k_\tau = 4.4\times10^{-6}$ m^{-1}. For $N = 2\Omega$ and $B_r = 5$G $(=0.45$ cm s$^{-1})$ we obtain $\omega_\beta = 7.8\times10^{-8}$ s^{-1} and $\omega_B = 2.8\times10^{-8}$ s^{-1}. The corresponding periods, $2\pi/\omega_\beta = 2.6$ years and $2\pi/\omega_B = 7.1$ years, are rather short.

Eq. (29) has two solutions, corresponding to the weakly decaying magnetic Rossby (MR) wave mode, $\overline{\omega}_{MR}$, and to the strongly decaying magnetic diffusion mode, $\overline{\omega}_{MR}$; the latter represents a skin-effect distorted by a fluid motion. To keep the algebra as simple as possible we write down here the solutions (only for $n=1$) using the small parameter $\omega_B^2/\omega_\beta^2 = 0.13(5/\text{m})^2$, and obtain the complex frequencies of the free modes in a very simple approximate form:

$$\overline{\omega}_{MR} = -\omega_\beta - \omega_\beta^2\omega_\beta^{-1} - i\gamma_{MR},$$

$$\gamma_{MR} = \tau_\eta^{-1}\omega_B^2\omega_\beta^{-2}, \tag{31}$$

$$\overline{\omega}_{MD} = \omega_B^2\omega_\beta^{-1} - i\gamma_{MD}, \quad \gamma_{MD} = \tau_\eta^{-1} \tag{32}$$

where $\tau_\eta^{-1} = \eta k_H^2$, $\tau_\eta \approx 10$ years. The simple expressions (31) and (32) for the two clearly separated modes are valid only for $n=1$. For $n>1$ the modes decay strongly and are not separated.

The main term in Eq. (31) is a well-known frequency of the Rossby waves, $-\omega_\beta$. These waves always propagate to the west with the non-dispersive phase velocity $V_\beta = \omega_\beta/k_\phi \approx 3.6$ cm s^{-1}. This velocity is much greater than fluid velocities in the core, and the latter may be ignored while considering magnetic Rossby waves. The Rossby frequency, ω_β, is non-dispersive because we neglected the inertia term $d_t(\nabla_\tau\times\boldsymbol{v}_\tau)_r$ in Eq. (18). The magnetic correction, $\omega_B^2\omega_\beta^{-1}$, in Eq. (31) is an order of magnitude smaller than ω_β, and it is highly dispersive. The decay rate, γ_R, of the MR waves is much smaller than even this magnetic correction. The group velocity of the weakly decaying MR waves can be calculated in a usual way. The energy of MR waves propagate to the west: $\partial\omega_{MR}/\partial k_\phi = V_\phi - V_\beta[1 + \omega_B^2\omega_\beta^{-2}k_\tau^{-2}(k_\phi^2 - k_\theta^2)]$. It is interesting to estimate different components of the energy density in the free MR waves, $\varepsilon_\Sigma = \varepsilon_\alpha + \varepsilon_b + \varepsilon_v$. For the energy components averaged over the volume the following proportions can be established:

$$\varepsilon_\alpha:\varepsilon_b:\varepsilon_v = \omega_\beta^2\left(k_\theta^2/k_\phi^2\right):\omega_B^2:\omega_\beta^2\left(k_\tau^2/k_H^2\right) \times (N/2\Omega_r)^2 \sim 4:10^{-1}:10^{-2} \tag{33}$$

The considered small-scale MR waves ($m\sim5$) have periods ~3 years, that is, one order of magnitude shorter than the axisymmetric SO waves. A simple (though not quantitatively valid) extrapolation shows that the global scale MR waves ($m = 1, 2$) have their periods in the decade range, ~30 years.

5. Topographic core–mantle coupling

Topographic coupling relies on the non-spherical form of the CMB, which can be described by the equation $r = R_1 - h(\theta, \phi)$. Here h gives the topography of the bottom of the SOC ($h>0$ for hills, and $h<0$ for valleys). The function $h(\theta,\phi)$ is unknown but it is certain that $h \ll R_1$, e.g., $h\sim0.3$ km corresponds to $h/R_1 \sim 10^{-4}$. The parallel to Ω component of the topographic torque, M_z, exerted on the mantle by the core can be written as

$$M_z = \rho_0\int sp\nabla_\phi h\,dA = -\rho_0\int sh\nabla_\phi p\,dA, \tag{34}$$

compare Roberts (1988), where the integral is taken over the CMB. Here $\rho_0 p$ is the pressure, dA is an element of the surface area. Braginsky (1998) estimated the torque, M_z, generated by interaction of the flow $V_\tau = 1_\phi V_\phi$ in the SOC with the topography, $h(\theta,\phi)$. A short derivation of this estimate is given below.

A topography $h(\theta,\phi)$ can be expressed as a sum of terms proportional to $\exp(i\boldsymbol{k}_\tau\cdot\boldsymbol{r}_\tau)$; their effects are mutually independent in the linear approximation. It should be noted that the effect of unevenness of the global wavelength, $\sim R_1$, is difficult to separate from the total core dynamics. We consider here only bumps of a rather short wavelength, $k_\tau R_1 \gg 1$. Their influence on the core motion can be considered as linear and local; then the plain model of the previous section is applicable. We assume the horizontal dependence of $h(x_\theta, x_\phi)$ to be the same as for the above magnetic Rossby waves: $h(x_\theta,\ x_\phi) = h_a \cos(k_\theta x_\theta)\exp(ik_\phi x_\phi)$, where h_a is the topography amplitude. The topography creates a velocity perturbation at the bottom of the SOC. To consider it we assume the same Eqs. (22), (23), (24a) and (24b), and the boundary conditions for velocity and magnetic field which were used for MR waves, with only one exception: the condition $v_r = 0$ at $r = R_1$ is replaced by $(\boldsymbol{V}_\tau + \boldsymbol{v}) \cdot \mathbf{1}_n = 0$, where $\mathbf{1}_n$ is a normal to the surface $h(x_\theta, x_\phi)$. In the linear approximation it takes the form

$$v_r(R_1) = -V_\phi \nabla_\phi h = -ihk_\phi V_\phi. \tag{35}$$

It is just the velocity perturbation Eq. (35) which generates the perturbation of pressure $p(R_1,\ \theta,\ \phi)$ and the topographic torque Eq. (34). Both the velocity perturbation Eq. (35) and the integrand of Eq. (34) are proportional to $\nabla_\phi h$; therefore, M_z is proportional to the amplitude of $(\nabla_\phi h)^2$. Note, however, that the integrals Eq. (34) would be zero if $p(R_1,\ \theta,\ \phi)$ were proportional to $h(\theta,\ \phi)$. Elementary solutions for v, b, etc., are sought proportional to $\exp(\alpha)$, where $\alpha = \kappa(r - R_1)$. Here $\kappa^2 = -k_r^2$, $\mathrm{Re}(\kappa) > 0$, where Re() denotes a real part, thus the perturbations generated by topography decrease with distance from the CMB. As a result, we obtain $\kappa^2 \approx \kappa_{\beta B}^2 = \kappa_\beta^2 + i\kappa_B^2$. We consider the stationary flow $\mathbf{1}_\phi V_\phi$, hence $\overline{\omega} = -k_\phi V_\phi$. The typical values $V_\phi \sim 5 \times 10^{-4}$ m s^{-1} and $k_\phi \sim 2.10^{-6}$ m^{-1} give $\overline{\omega} \sim 10^{-9}$ s^{-1}. The quantities $\kappa_\beta^2 = k_H^2 \omega_\beta / \overline{\omega}$ and $\kappa_B^2 = \omega_B^2 / \eta\overline{\omega}$ arise in Eq. (29). Using the same values of parameters as in Section 3 we obtain the estimates $\kappa_\beta H \sim 28$ and $\kappa_B H \sim 50$. We are interested in the 'topographic stress', that is, in the averaged value of the tangential force per unit area, $\pi_{r\phi}^{\mathrm{h}}$. Using the integrand of M_z, and $\nabla_\phi p = -2\Omega_r v_\theta$, we can write $\pi_{r\phi}^{\mathrm{h}} = -\rho_0 \langle h \nabla_\phi p \rangle = 2\Omega_r \rho_0 \langle h v_\theta \rangle$. Here v_θ can be expressed through v_r, like in the MR waves, and v_r can be connected with $h(x_\theta, x_\phi)$ by Eq. (35). Transforming all quantities to the real form and averaging $\mathrm{Re}(h)\mathrm{Re}(v_\theta)$ over x_θ, x_ϕ, we obtain

$$\pi_{r\phi}^h = \rho_0 (N h_a / 2)^2 \left(k_\phi / |\kappa_{\beta B}|\right) \sin\varphi_{\beta B}, \tag{36}$$

where $|\kappa_{\beta B}|^2 = (\kappa_B^4 + \kappa_\beta^4)^{1/2}$, $\sin(2\varphi_{\beta B}) = \kappa_B^2 / |\kappa_{\beta B}|^2$. Here $\sin\varphi_{\beta B}$ is a numerical factor determined by a phase difference between h and v_θ; it has the same sign as V_ϕ. For example, $\sin\varphi_{\beta B} = 0.64$, 0.59, 0.49, and 0.38 for $\kappa_\beta^2 / \kappa_B^2 = 0.2$, 0.3, 0.6, and 1. For the parameters used above ($m = 5$, etc.), and for $\sin\varphi_{\beta B} \sim 0.5$ we have

$$\pi_{r\phi}^h \sim 8.5 \times 10^{-2} h_a^2 \left(V_\phi / V_{\phi_0}\right)^{1/2}, \tag{37}$$

where $\pi_{r\phi}^{\mathrm{h}}$ is expressed in N m^{-2}, h_a in km, and $V_{\phi 0} = 5 \times 10^{-2}$ cm s^{-1}.

To interpret Eq. (36), we note that the fluid stream near the bottom of the SOC has to cross the hill, h_a, therefore, it moves up the slope $k_\phi h_a$, against the retarding component (i.e., component parallel to the slope) of gravitational force $\rho_0 g_{\mathrm{ret}} \sim \rho_0 g_r (h_a \partial_r C) k_\phi h_a \sim \rho_0 k_\phi h_a^2 N^2$. It is assumed here that $c \sim h_a \partial_r C$. The retarding force acts in the moving fluid layer of thickness $\sim |\kappa_{\beta B}|^{-1}$, thus creating the stress of order of Eq. (36). The retardation is partially compensated when the stream goes down the slope, but owing to the lack of the 'for-and-aft' symmetry of the solution, the non-compensation is left, which is measured by the coefficient $\sin\varphi_{\beta B}$.

It is interesting to compare Eq. (36) with magnetic stresses due to the finite conductivity of the mantle, σ_{M}. It can be easily estimated if we assume that the mantle conductivity is concentrated in the narrow layer near the CMB. The electric current of density $j_\theta = \sigma_{\mathrm{M}} V_\phi B_{\mathrm{r}}$ flowing in the layer of small thickness, L_{M}, generates the magnetic stress $j_\theta B_{\mathrm{r}} L_{\mathrm{M}}$, equal to

$$\pi_{r\phi}^B = \sigma_{\mathrm{M}} L_{\mathrm{M}} B_{\mathrm{r}}^2 V_\phi \sim 3.8 \times 10^{-3} \left(V_\phi / V_{\phi 0}\right). \tag{38}$$

Here, the estimate $\sigma_{\mathrm{M}} L_{\mathrm{M}} \sim 3 \times 10^7$ S of the lower mantle conductivity is taken according to the results by Peyronneau and Poirier (1989). The topographic stress Eq. (37) is greater than the magnetic one if the amplitude of the unevenness h_a is greater than ~ 0.2 km. The perturbation produced by the CMB topography is considered here in a linear approximation, which is valid for $k_\theta v_\theta / k_\phi V_\phi \ll 1$. This ratio can be estimated as $k_\theta v_a / k_\phi V_\phi \sim h_a (V_{\phi 0} / V_\phi)^{1/2}$, where h_a is expressed in km. For example, at $h_a \sim 0.2$ km the linearization is still valid.

6. Waves of diurnal periods

Three kinds of waves with the frequencies $\sim \Omega$ and greater are well-known in the physics of the common ocean: (1) gyroscopic (inertial) waves, which can propagate even in the homogeneous fluid without any elastic forces, (2) internal gravity waves, which gain their 'elasticity' from the density stratification of the fluid, and (3) internal surface gravity waves, running on the surfaces of a density jump (see, e.g., Gill, 1982; Brekhovskikh and Goncharov, 1985; Cushman-Roisin, 1994). All these kinds of waves are possible in the fluid core with the SOC. They can be considered qualitatively by replacing the spherical SOC with a simple plane model which is often employed in the theory of rotating fluids.

It is easy to see that the magnetic force, $\boldsymbol{f}^B$, is relatively small in these oscillations. The term $\eta\nabla^2\boldsymbol{b}$ in Eq. (4) is much smaller than $\partial_t\boldsymbol{b}$ for $\omega\sim\Omega$. Neglecting it and writing $\partial_t\boldsymbol{b}=(\boldsymbol{B}\cdot\nabla)\boldsymbol{v}$, we obtain $\partial_t\boldsymbol{f}^B=(\boldsymbol{B}\cdot\nabla)\partial_t\boldsymbol{b}=(\boldsymbol{B}\cdot\nabla)^2\boldsymbol{v}$. The force, $\boldsymbol{f}^B$, is much smaller than the Coriolis force, $\boldsymbol{f}^{\Omega}\sim 2\boldsymbol{\Omega}\times\boldsymbol{v}$. Assuming $\boldsymbol{B}\cdot\nabla\sim B_r\pi/H$ we have an estimate $f^B/f^{\Omega}\sim(B_r\pi/H)^2/2\Omega\omega$. For $\omega\sim\Omega$, $B_r\sim 5$ G $(=0.45$ cm s$^{-1})$ and $H=80$ km, this ratio is $\sim 3\times 10^{-6}$. The effect of magnetic field can be ignored when the frequencies and forms of oscillations are considered. It is, however, of utmost significance for the decay rate of the waves because the Joule heat production is the main mechanism of dissipation, while Archimedean and Coriolis forces are dissipationless.

A contained rotating fluid is prone to waves called inertial (Greenspan, 1969) or gyroscopic (Brekhovskikh and Goncharov, 1985). In the inertial waves there is an interplay between the inertia force, $f^{\mathrm{I}}\sim\omega v$, and the Coriolis force, $f^{\Omega}\sim 2\Omega v$, hence $f^{\mathrm{I}}\sim f^{\Omega}$ and $\omega_{\mathrm{I}}\sim\Omega$. The solution for these waves in the sphere filled with an incompressible nonviscous fluid was obtained analytically, and it reveals an infinity of modes of increasingly complicated space structure (see Greenspan, 1969). Solutions for more geophysically relevant case of the fluid sphere, which contains the solid inner core, were obtained numerically by Rieutord (1995). Inertial waves develop in the whole volume of the fluid core. The SOC occupies only about 7% of the core, therefore, its influence on ω_{I} is weak. The SOC influence on the fluid motion is not negligible, however; the large difference between stratification in the SOC and in the bulk of the core results in a jump of the tangential velocity of oscillation, $[\![\boldsymbol{v}_\tau]\!]$, at the SOC surface, $r=R_{\mathrm{S}}$.

Internal waves in rotating fluids have also been extensively investigated in oceanology and meteorology. Plane layer models employed in Chapter 11 of Brekhovskikh and Goncharov (1985), and in Chapter 8 of Gill (1982) are also applicable for consideration of the waves in the SOC because magnetic force in the core is negligible for frequencies $\omega\sim\Omega$. Similar calculations using a simple plane model and the boundary conditions (7) and (9) specific for the SOC shows that the internal waves can exist, which are mostly concentrated in the SOC, while the main part of the core is nearly undisturbed. This localization is a result of a very large N_{S}^2, which is much greater then N^2, and Ω^2. The large jump of fluid density on the SOC boundary acts like a nearly rigid wall confining the motion inside the SOC. The calculations show that the wave frequency is determined mostly by combination $\Omega_r^2+N^2(k_\tau/k_r)^2$, but in the large-scale waves $k_\tau/k_r\ll 1$, therefore the role of stratification is small inside the SOC. One may consider these internal waves as inertial waves trapped in the SOC because of a large N_{S}^2. The frequencies of both inertial and internal waves are of the same order of magnitude, $\omega\sim\Omega_r$ (in the interval from Ω to 2Ω), and the periods about 16 h may be expected for both of them. Kinetic energy is the main energy component of both inertial and internal waves, but the latter occupies much smaller volume than the former, therefore, at the equal amplitude of velocity the total energy of internal waves is about one order of magnitude smaller than that of the inertial waves.

As is well known, gravitational waves can propagate at a density jump inside a fluid, like the common surface waves do at the external fluid surface. Similarly, the waves concentrated near the surface, $r=R_{\mathrm{S}}$, can propagate in the SOC. If $k_\tau R_1\gg 1$ then the waves are essentially local and can be described adequately by the plane layer model. For the case of a large horizontal wave number, $k_\tau H\gg 1$, which is called 'deep water approximation' in oceanology, the frequency of the surface waves in a non-rotating and non-stratified fluid is $\omega_g=(gC_{\mathrm{S}}k_\tau/2)^{1/2}=N_{\mathrm{S}}(k_\tau H/2)^{1/2}$ (see, e.g., Landau and Lifshitz, 1987). Substituting $N_{\mathrm{S}}\sim 20\Omega$ and $k_\tau H\sim 5$ we obtain $\omega_g\sim 30\Omega$, hence rotation and stratification are negligible indeed. The opposite case, $k_\tau H\ll 1$ (but $k_\tau R_1\gg 1$), is called 'shallow water approximation' in oceanology. In this case the approximate expression for frequency is $\omega = N_S k_\tau H$, and the correction due to rotation and stratification contains the small terms $\sim k_\tau H$ and $\sim (\Omega/\omega)^2$. If $N_S \sim 10N \sim 20\Omega$, and $k_\tau H = 0.2$ then $\omega \sim 4\Omega$. The main part of the energy of the surface waves is given by the term $\varepsilon_S = N_S^2 h_S^2/2$ in the energy balance Eq. (11).

An energy dissipation and decay of the waves is determined by Joule heat production due to the motion of conducting fluid across the magnetic field of the core. The simplest way to estimate the decay rate of the wave is to find the electric current, $\boldsymbol{j} = \nabla \times \boldsymbol{b}$, generated by the wave motion and calculate Joule heat production; then the energy balance can be used. The magnetic field, b, can be calculated by means of Eq. (4), using the fluid velocity, v, given by the equation of motion with the magnetic force omitted. All oscillating quantities (v, b, c, p) are proportional to $\exp(-\gamma t)$. According to the energy balance, we have

$$\gamma = Q_J/2E_\Sigma. \tag{39}$$

The main part of Q_J is produced in the thin electromagnetic 'skin-layers'. If δ is the thickness of any current layer, and the change of magnetic field across the layer is $\sim \Delta b$, then the current in the layer is of order of $j \sim (\Delta b/\delta)$, and the integral of the Joule heat over the layer is proportional to $j^2\delta \sim (\Delta b/\delta)^2\delta \sim (\Delta b)^2/\delta$. The thinner the layer the larger is the total dissipation of energy for a given Δb. Thin electromagnetic skin-layers can develop on the surfaces where the velocity of the fluid oscillating across magnetic field lines experiences a jump. We call such fluid motion 'the magnetic field lines cutting'. The skin depth, $\delta_\eta = (2\eta/\omega)^{1/2}$, is very small for the period of one day: $\delta_\eta(\Omega) \approx 2.3 \times 10^2$ m. The magnetic diffusivity of the core, $\eta = 2$ m^2 s^{-1}, is taken here according to Braginsky and Roberts (1995).

The magnetic field lines cutting mechanism can be easily considered in the case of a simple plane geometry. The constant homogeneous magnetic field is $\mathbf{1}_r B_r$, and the oscillating velocities, $\mathbf{1}_y v_y^+$ and $\mathbf{1}_y v_y^-$, are homogeneous above and below the plane $r = r_0$, where the diffusivities are η_+ and η_-. The velocity jump is $[\![v_\tau]\!] = v_y^+ - v_y^-$. This motion 'cuts' the magnetic field lines of the field $\mathbf{1}_r B_r$. Eq. (4) can be easily solved in this case, and integrating ηj^2 by dr over the skin-layer we obtain the following expression for the Joule dissipation (divided by ρ_0) per unit of the area of the surface $r = r_0$:

$$Q_J^{\text{cut}} = B_r^2 \left[(\eta_+ + \eta_-)\,\omega \right]^{-1/2} [\![v_\tau]\!]^2/2 \tag{40}$$

Two extreme cases are of the main interest: (a) $\eta_+ = \eta_- = \eta$, valid at $r_0 = R_S$ (the ocean's surface), and (b) $\eta = \eta_- \ll \eta_+ = \eta_M$, valid at $r_0 = R_1$. Here η_M is the mantle magnetic diffusivity at the CMB determining the core–mantle friction, Q_J^M. These results can be written as

$$Q_J^S = B_r^2 [\![v_S]\!]^2 \left(2\,\omega \delta_\eta \right)^{-1}, \tag{40a}$$

$$Q_J^M = B_r^2 v_1^2 \left(2^{1/2} \omega \delta_{\eta M} \right)^{-1}, \tag{40b}$$

where $\delta_{\eta M} = (2\eta_M/\omega)^{1/2}$. Here v_1 is the fluid velocity at $r = R_1$ relative to the mantle. Comparing these estimates, we obtain $Q_J^S/Q_J^M = ([\![v_S]\!]/v_1)^2 \times (\eta_M/2\eta)^{1/2}$. If $[\![v_S]\!] \sim v_1$ then $Q_J^S/Q_J^M \sim (\eta_M/\eta)^{1/2} \sim 10^2$. It can be shown that in an inertial wave the velocity jump at $r \equiv R_S$ is of order of the wave amplitude, $[\![v_S]\!] \sim v_a$, if the quantity $a_S = (k_\tau H N_S/2\Omega)^2$ is of order of unity or greater. This implies a relatively large Joule dissipation and decay rate of the waves.

7. Concluding remarks: how to prove the existence of the SOC

The SOC model (1) contains three parameters: H, C_H, and C_S. The first pair, H and C_H, was estimated in Braginsky (1993) by comparing an approximate theory of the 65-year oscillation with approximate values of the observed oscillations amplitudes, g_1^0 and Λ_d. The order of magnitude estimate, $C_S \sim 10^2 C_H$, of the third parameter follows from the modern seismic data as a plausible assumption. If the existence of the SOC is proven then the picture of the two-component Earth's core will be replaced with the three-component one, and a new branch of geophysics will be established to investigate the SOC specific oceanology. To accept such a serious consequences we have to be certain that the SOC definitely exists and its model is adequate. A fearsome principle of the Occam razor ('It is vain to do with more what can be done with less') prohibits introduction of a new entity except if it is unavoidable.

How to prove (or disprove) the very existence of the SOC? The regular way to do this is to find a large enough number of various observed effects, and to compare the values of these effects—let us call them the 'observed parameters' (OP), with the theoretically calculated values of the same effects expressed through the 'unknown parameters' (UP). The UP include the above three parameters of the SOC, and maybe also artificially added fitted parameters of the theory. If the number of the OP is significantly greater then the number of the UP, and if the cross-check gives the same values of the UP for all tests (with a reasonable accuracy) then we are certain that the SOC does really exists at the top of the core, and we know the wanted parameters. Otherwise something is wrong in our understanding of the situation. Here is a list of various kinds of relevant observational data, followed by comments.

1. The seismic data.
2. The decade l.o.d. and geomagnetic variation data, especially on the 65- and ≈ 30-year oscillations.
3. The superconducting gravimetry data on the diurnal core oscillations.
4. The data on the Earth orientation, especially from the very large base interferometry (VLBI).

(1) Identification of the SOC by seismological methods is hampered by obscuring influence of the D″ layer in the mantle nearby. The SmKS rays have a significant part of their paths in the outermost core where they experience $m-1$ reflections from the CMB, that is why these phases are used to investigate the seismic properties of the core just below the CMB. The differential travel times, Δ, of the SmKS are especially sensitive to seismic velocity in the top of the core, but they are also dependent on the D″ layer inhomogeneity. Sylvander and Souriau (1996) calculated about a thousand of these differential travel times: $\Delta(1)$ for S2KS-SKS and $\Delta(2)$ for S3KS-S2KS. Both $\Delta(1)$ and $\Delta(2)$ have a dispersion due to the D″ inhomogeneity, and they expose also a systematic shift, $\langle \Delta \rangle$ relative to the PREM model. According to Sylvander and Souriau (1996) ''two possibilities may account for the $\langle \Delta(1) \rangle$ as well as the $\langle \Delta(2) \rangle$: a global decrease in the D″ average shear-wave velocity, or lower velocities in the outermost core''. The second possibility corresponds to the existence of the SOC. The model (1), having $\sim 1\%$ jump in material properties (density, seismic velocity) at the SOC surface, $r = R_S$, gives an effective method to solve this dilemma. The seismic waves can be reflected from the sharp SOC surface at $r = R_S$; this effect is not obscured by the D″ layer and can be used for the SOC diagnostic. Seismic wave experiences total reflection from the surface of the jump in wave velocity, if the angle of incidence of the wave is close enough to 90°; it should be closer than $\approx 8°$ for $\sim 1\%$ velocity jump. The effects of reflections of seismic rays from the surface $r = R_S$ will hopefully be found in the seismic records, then H and C_S can be estimated.

(2) A rough theory of the ω_1 mode of the SOC oscillations, and two observed magnitudes, Λ_d and g_1^0, were used in Braginsky (1993) to estimate two parameters of the SOC, H and N. To obtain the number of OP significantly greater than the number of UP we have to develop a quantitative theory for both ω_1 and ω_2 $(\approx 2\omega_1)$ modes together with associated torsional oscillations and Rossby waves. To solve this difficult problem of the complex set of non-linearly interacting oscillations we have to understand their mechanisms of excitation. One candidate could be the (properly parameterized) baroclinic instability mechanism, which is well known in oceanology and meteorology. With the decade oscillation theory at hand it will be possible to calculate the decade l.o.d. variations (from angular momentum balance), and find the decade geomagnetic variations. The l.o.d. oscillations are known from observations with good precision, therefore, amplitudes and phases of both modes of oscillations can be calibrated by fitting the calculated l.o.d. to their observed values. With such a theory the comparison of theoretical and observed geomagnetic decade variations could (hopefully) include the Gauss coefficients of the geomagnetic variation, up to $n,m = 2$ (or may be even 3), for both ω_1 and ω_2 modes. This would make it possible to cross-check the theory against observation with the number of OP greater than the number of UP. The oscillating core–mantle torque and the corresponding coefficients of friction on the CMB also can be estimated for two frequencies, ω_1 and ω_2. This will help to better understand the mechanism of the core–mantle coupling.

The modern techniques (superconducting gravimetry and VLBI, which are very precise) provide us with a new data, which are sensitive to the variations in the Earth inner motion, and in the Earth orientation. These data also can be used for the SOC diagnostic.

(3) Aldridge and Lumb (1987) tentatively identified, in the superconducting gravimeter records of two earthquakes, several oscillations in the period interval of 13–16 h, which they attributed to inertial waves in the fluid core. According to Hinderer (1997), this core mode identification was not confirmed by the later investigations. Nevertheless, one can hope that a new mode identification will be obtained with accumulation of more data from superconducting gravimeters. According to the discussion in Section 6 the SOC inner waves with periods in the same interval are also possible. It seems plausible that the inner waves, which are localized in the SOC, close to the mantle, have more chances to be identified. These waves decay due to a rather strong Joule heating mechanism. The accurate values of the inner waves frequencies and their decay rates are sensitive to the SOC parameters, and can be used for the SOC diagnostic.

(4) The real and imaginary parts of the Earth's nutations amplitudes, as well as frequency and decay rate of the Chandler wobble depend on the processes in the SOC. Corresponding corrections to existing theories of these effects can be calculated, and their comparison with VLBI data may give us additional opportunities to cross check the SOC parameters.

Acknowledgements

This work was supported by NASA Grant NAG 5-2953. I am grateful to Dr. Thorne Lay for an illuminating discussion of the modern seismic investigations of the top of the core, and to two anonymous referees for useful comments.

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