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# Effect of the stratified ocean of the core upon the Chandler wobble

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**Abstract**

It is supposed that a stably stratified layer exists at the top of the fluid core; we call it the stratified ocean of the core (SOC). The assumed model of the SOC has a sharp density drop at the boundary with the bulk of the core, and a uniform internal density gradient. The influence of the SOC upon the Chandler wobble is considered. It is shown that the action of Archimedean buoyancy forces in the SOC creates inside this layer an additional fluid velocity of order of $(C_H/2\sigma)v_\tau$. Here $C_H$ is the relative density deficit in the SOC, $v_\tau$ is the velocity associated with the Chandler wobble without the SOC, and $\sigma \sim 1/400$ is the frequency of the Chandler wobble measured in cycles per day. The effect of the SOC on the Chandler wobble is small: it increases the wobble period by $\sim 0.1$ day, and its contribution to the dissipation of the wobble energy is about six orders of magnitude smaller than the observed dissipation. A change of seismic velocity in the SOC due to the greater concentration of the light admixture is discussed. A comparison of the conditions in the SOC with the conditions at the inner core boundary shows that the extra amount of the light fluid in the layer increases the seismic velocity; therefore, the SOC is a high-velocity layer. 

*Keywords:* Chandler wobble; Earth core; Stably stratified layer

## 1. Introduction

The assumption that the top of the fluid outer core of the earth, adjacent to the core–mantle boundary (CMB), is stably stratified has been discussed for a long time, see Whaler (1980), Fearn and Loper (1981), Yukutake (1981), Frank (1982), Gubbins et al. (1982), Braginsky (1984), Bergman (1993), Braginsky (1993), Braginsky and Le Mouël (1993), Loper and Lay (1995), Shearer and Roberts (1997), Braginsky (1998), Lister and Buffett (1998), Braginsky (1999). There is a close similarity between this stably stratified layer at the top of the core and the earth's oceans, both in their thin shell geometry and in the magnitude of their Brunt-Väisälä frequencies. That is why we call it the **s**tratified **o**cean of the **c**ore (SOC). The SOC is characterized by its (negative) density excess, $C = (\rho - \rho_a)/\rho_a$. Here $\rho$ is the true fluid density, and $\rho_a$ is the density corresponding to the adiabatic gradient. Braginsky (1993) assumed the following three-parameter model of the SOC with the equilibrium density excess, $C_0$, having the linear radial dependence:

$$C_0 = -C_S - (C_H/H)(r - R_S). \qquad (1)$$

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Braginsky (1993) has shown that two observed effects can be explained by the same physical process in this stably stratified layer, namely that the 65-year variations of both the geomagnetic field and the speed of the earth's rotation can be generated by an axially symmetric oscillation in the SOC, akin to a MAC-wave. The two main parameters of the layer were estimated by Braginsky (1993) by comparing the theory of the 65-year oscillation, with the observed variations of the geomagnetic dipole and of the length of day. These two parameters are the layer's thickness, $H \approx 80$ km, and the Brunt-Väisälä frequency, $N = (g_1 C_H / H)^{1/2} \approx 2\Omega$, which corresponds to $C_H \sim 10^{-4}$. Here $\Omega = 0.729 \times 10^{-4}$ s$^{-1}$ is the angular velocity of the earth, and $g_1 = 10.68$ m s$^{-2}$ is the acceleration due to gravity at the CMB. The CMB situated at radius $R_1$ is the bottom of the ocean, and its surface merges with the nearly **a**diabatic **b**ulk of the fluid **c**ore (ABC) at radius $R_S = R_1 - H$, where the layer's density excess, $C_0$, drops to a very small value, $C_{ABC} \sim 10^{-8}$ or smaller. This very small value is ignored in the present paper.

Several investigators (Lay and Young, 1990; Souriau and Poupinet, 1991; Garnero et al., 1993; Sylvander and Souriau, 1996; see also a recent paper by Garnero and Lay, 1998) suggested that a low-velocity layer about 50–100 km thick with a $\sim 1\%$ diminution in seismic velocity exists in the uppermost core. This effect is not reliably measured, and an alternative explanation of the relevant observations was suggested, which is based on processes in the D″ layer in the mantle, see, e.g., Sylvander and Souriau (1996). If a $\sim 1\%$ change in seismic velocity exists in the SOC then one should assume that $C \sim 10^{-2}$ there also, and the only way to avoid a contradiction with the results of Braginsky (1993) is to assume a jump $C_S \sim 10^{-2}$ at the SOC surface, $r = R_S$, with the density gradient inside the layer determined by the much smaller value $C_H$. These reasons led Braginsky (1998, 1999) to the assumption $C_S \sim 10^{-2} \sim 10^2 C_H$.

It should be noted, however, that the very existence of the low velocity layer at the CMB is questionable. The seismic velocity in the fluid core is $v_P = (K/\rho)^{1/2}$, where $K$ is the adiabatic incompressibility, hence $2\Delta v_P / v_P = -(\Delta\rho/\rho)(1-\kappa)$, where $\kappa = (\Delta K/K)/(\Delta\rho/\rho)$; small changes are designated by $\Delta$. If both $\Delta K$ and $\Delta\rho$ are proportional to a change in the light admixture concentration, $\Delta\xi$, then the ratio $\kappa$ does not depend on $\Delta\xi$. Assuming that the light material arrives from the inner core we estimate $\kappa$ from the changes $\Delta K$ and $\Delta\rho$ at the inner core boundary where a jump of $\xi$ exists. From the PREM model we find $\Delta K/K \approx 0.03$ and $\Delta\rho/\rho \approx 0.05$; thus $\kappa \approx 0.6 < 1$. Hence, $\Delta v_P / v_P$ and $\Delta\rho/\rho$ have opposite signs, and $\Delta v_P > 0$ in the SOC. The relation $\Delta v_P / \Delta\rho < 0$ is quite common when the density changes due to the change of composition, see Anderson (1995), Fig. 5.11. The SOC is not a low velocity layer with $\Delta v_P / v_P \sim 10^{-2}$, and the reasons of Braginsky (1998, 1999) for assuming a large jump, $C_S$, at the surface of the SOC are not valid. A jump $C_S$ may or may not exist. An attempt was made by Braginsky and Earle (1999) to find the reflections of seismic waves from the density jump on the SOC surface. These reflections were not found, but an upper bound was estimated for the magnitude of the jump: $C_S < 10^{-3}$.

To understand the effect of the SOC on the Chandler wobble one should consider the accompanying core fluid motion. If the fluid inside the SOC moves together with the ABC then a significant effect cannot be expected. If, however, the SOC were to stick to the mantle, then the mantle's effective moment of inertia increases by $\sim 1\%$, which would produce an increase in the Chandler period, $T_{CW}$, of several days. It should be noted that the observed value, $T_{CW} = 435$ days, is obtained theoretically by assuming that the earth's anelasticity grows with the decrease in frequency of oscillations, and this increases $T_{CW}$ by $\sim 8$ days; see Smith and Dahlen (1981), and Molodenskiy and Zharkov (1982). A consideration of the rheological properties of Earth materials by Anderson and Minster (1979) confirmed the existence of such an anelasticity dependence. If an increase in $T_{CW}$ of several days were ascribed to the SOC then our understanding of the earth's rheology would be questioned. This makes it especially interesting to investigate in detail the fluid motion in a wobbling ellipsoid that has a light fluid layer at its top. It was noted by Toomre (1974) that ''not even understood properly ... is the idealized example where the rotating core consists of just two incompressible gravitating fluids of different uniform densities, with the denser liquid of course residing deeper inside''. The present paper begins with a general model but the final results are obtained for this idealized example.

The basic assumptions of this paper are mostly the same as those of previous works on the theory of the earth's wobble and nutation; see Molodensky (1961), Sasao et al. (1980), etc. The only new terms added to the equations of core dynamics and to the boundary conditions are those connected with a specified Archimedean force acting on the SOC.

## 2. Main equations

In a basic state of hydrostatic equilibrium the earth rotates around the $z$-axis with a constant angular velocity $\boldsymbol{\Omega} = \mathbf{1}_z \Omega$. We use a reference system fixed to the mantle, which rotates with the angular velocity $\boldsymbol{\Omega}^{\mathbf{m}} = \Omega[\mathbf{1}_z + \mathbf{m}(t)]$ perturbed by the variations in the earth's orientation, e.g., the Chandler wobble (CW) or the earth's nutations; here $\mathbf{m}(t)$ is the customary notation for this small perturbation. The mantle's mean principal axes are $(x, y, z)$, the corresponding unit vectors are $\mathbf{1}_x$, $\mathbf{1}_y$, $\mathbf{1}_z$. Spherical coordinates $(r, \theta, \phi)$ and cylindrical coordinates $(z, s, \phi)$ are used, the corresponding unit vectors are $\mathbf{1}_r$, $\mathbf{1}_\theta$, $\mathbf{1}_\phi$, and $\mathbf{1}_s$. The obvious relations, $\mathbf{r} = z\mathbf{1}_z + \mathbf{s}$, $r^2 = z^2 + s^2$, $z = r \cos\theta$, $s = r \sin\theta$, $x = s \cos\phi$, $y = s \sin\phi$, are used below. The vector $\mathbf{m}(t) = \mathbf{1}_x \mathbf{m}_x(t) + \mathbf{1}_y \mathbf{m}_y(t)$ rotates about the $z$-axis with a constant angular frequency $\omega$, hence $d_t \mathbf{m} = \omega \mathbf{1}_z \times \mathbf{m}$ and $d_t^2 \mathbf{m} = -\omega^2 \mathbf{m}$. We neglect fluid viscosity and magnetic forces; hence the core dynamics is governed by the well known hydrodynamic equation of motion, which in the frame rotating with velocity $\boldsymbol{\Omega}^{\mathbf{m}}$ takes the form:

$$(\partial_t + \boldsymbol{v} \cdot \nabla)\boldsymbol{v} + 2\boldsymbol{\Omega}^{\mathbf{m}} \times \boldsymbol{v} + d_t \boldsymbol{\Omega}^{\mathbf{m}} \times \boldsymbol{r} = -\rho^{-1} \nabla p - \nabla U_e. \quad (2)$$

Here $\mathbf{v}$, $\rho$, and $p$, are the fluid velocity, density, and pressure, $U$ is the gravitational potential, and $U_e = U - (\boldsymbol{\Omega}^{\mathbf{m}} \times \mathbf{r})^2/2$ is an effective potential (including a centrifugal potential). It is assumed that the equilibrium surfaces of constant density, pressure, and effective potential are coincident oblate ellipsoids of oblateness $\varepsilon$. They are described by the expression:

$$r = r_e[1 - \varepsilon(r_e)(\mu^2 - 1/3)], \quad (3)$$

where $\mu = \cos\theta$, and $r_e$ is the average radius of the ellipsoidal surface. The core-mantle boundary (CMB) is given by $r_e = R_1$, and the ABC-SOC boundary corresponds to $r_e = R_S = R_1 - H$. The equilibrium pressure, density, and density excess, as well as the effective potential bear the subscript zero, and depend only on $r_e$: $p_0(r, \theta) = p_0(r_e)$, $\rho_0(r, \theta) = \rho_0(r_e)$, and $U_0(r, \theta) - (\boldsymbol{\Omega} \times \mathbf{r})^2/2 = U_{e0}(r_e)$. Therefore the equation of hydrostatic equilibrium can be written in the vector, or the equivalent scalar, forms as:

$$\rho_0^{-1} \nabla p_0 = \boldsymbol{g}, \quad \rho_0^{-1} \partial_n p_0 = -\partial_n U_{e0} = g_n. \quad (4a,b)$$

Here $\mathbf{g} = -\nabla U_{e0}$, and $\partial_n = \mathbf{1}_n \cdot \nabla$ is the derivative along the normal to an equilibrium ellipsoidal surface: $\mathbf{1}_n = \mathbf{1}_r - \varepsilon \mathbf{1}_\theta \sin 2\theta$. Strictly, one should replace $r$ by $r_e$ in Eq. (1).

We consider small linear oscillations of the earth, and assume $\rho = \rho_0 + \rho_v$, $p = p_0 + p_v$, $U = U_0 + U_v$. The quantities with the subscript $_v$, as well as $\mathbf{m}(t)$ and $\mathbf{v}$, are small variations due to the CW or nutation. They experience linear harmonic oscillations of frequency $\omega$; a non-dimensional frequency $\sigma = \omega/\Omega$ is also used below.

The density depends on pressure, $p$, entropy, $S$, and admixture concentration, $\xi$. The gradient of the equilibrium density, $\rho_0(p_0, S_0, \xi_0)$, and the density variation, $\rho_v$, can be written as:

$$\nabla \rho_0 = v_P^{-2} \nabla p_0 + \rho_0 \nabla C_c, \quad \rho_v = v_P^{-2} p_v + \rho_0 C_v, \quad (5a,b)$$

where

$$\nabla C_c = -\alpha^S \nabla (S_0 - S_a) - \alpha^\xi (\xi_0 - \xi_a),$$

$$C_v = -\alpha^S S_v - \alpha^\xi \xi_v. \quad (6a,b)$$

Here $v_P = (\partial p_0 / \partial \rho_0)^{1/2}$ is the velocity of seismic P-waves in the fluid core. The adiabatic quantities, $S_a$ and $\xi_a$, are constant inside the ABC and inside the SOC. We assume the entropy and admixture expansion coefficients, $\alpha^S = -\rho_0^{-1} \partial \rho_0 / \partial S_0$ and $\alpha^\xi = -\rho_0^{-1} \partial \rho_0 / \partial \xi_0$, to be constant for simplicity; hence the equilibrium non-adiabatic density excess is $C_c = -\alpha^S (S_0 - S_a) - \alpha^\xi (\xi_0 - \xi_a)$. We accept $S_0 = S_a$ and $\xi_0 = \xi_a$ (hence $C_0 = 0$) inside the ABC, neglecting $C_c \sim 10^{-8}$ and $C_v$ there. We put $S_a = S_0(R_S + 0)$ and $\xi_a = \xi_0(R_S + 0)$. According to model (1) we have inside the SOC $C_0 = -C_S + C_c$, where $C_c = -(C_H/H)(r_e - R_S)$, $C_c(R_1) = -C_H \sim 10^{-4}$, and at the ABC-SOC boundary there is a density jump $[[\rho_0]] = [[\rho_a]] = -\rho_0 C_S$; we assume $C_S \sim 10^{-3}$.

The density change inside the SOC, due to advection of entropy and admixture along their gradients, $\nabla S_0$ and $\nabla \xi_0$, can be written in the linear approximation as

$$\partial_t C_v = -\boldsymbol{v} \cdot \nabla C_c = -v_n N^2/g_n,$$

$$N^2 = \mathbf{g} \cdot \nabla C_c \approx g_1 C_H/H. \qquad (7a,b)$$

Here $N$ is a Brunt–Väisälä frequency, and $-g_n \approx g_1 = 10.68$ m/s$^2$ in the SOC. Eq. (7a,b) for the oscillating variations can be integrated: $C_v = -\mathbf{u} \cdot \nabla C_c = -u_n \partial_n C_c$, where $u_n$ is the fluid displacement normal to equilibrium surfaces, $v_n = \partial_t u_n$.

The linearized hydrodynamic equation for the small variations takes the form:

$$\partial_t \boldsymbol{v} + 2\boldsymbol{\Omega} \times \boldsymbol{v} = -\nabla\psi + \boldsymbol{f}^w + \boldsymbol{f}^N, \qquad (8)$$

where

$$\psi = \rho_0^{-1} p_v + U_v + \psi_w, \quad \psi_w = (\Omega + \omega)\Omega z(\boldsymbol{s} \cdot \boldsymbol{m}), \qquad (9a,b)$$

$$\boldsymbol{f}^w = \mathbf{1}_z 2\Omega\omega(\boldsymbol{s} \cdot \boldsymbol{m}),$$

$$\boldsymbol{f}^N = \boldsymbol{g} C_v - p_v \rho_0^{-1} \nabla C_c$$

$$= -1_n N^2 \left(u_n + p_v \rho_0^{-1} g_n^{-1}\right). \qquad (10a,b)$$

Here $\mathbf{s} \cdot \mathbf{m} = \mathbf{r} \cdot \mathbf{m}$; The pressure gradient variation, $\rho^{-1}\nabla p - \rho_0^{-1}\nabla p_0 = \nabla(\rho_0^{-1} p_v) - \mathbf{f}^N$, includes (only in the SOC) the term (10b), $\mathbf{f}^N$, proportional to $N^2$. The second term in Eq. (10a,b) is usually neglected, see e.g., Smylie and Rochester (1981), but it may be comparable with the first term if $u_n$ is very small. The inertial force $\mathbf{f}^w$ is due to the mantle wobbling; it provides the excitation of the core's fluid motion.

The mass conservation equation will be written in the anelastic approximation: $\nabla \cdot (\rho_0 \mathbf{v}) = 0$; this can be rewritten as:

$$\nabla \cdot \boldsymbol{v} = k_\rho v_n, \quad k_\rho = -\rho_0^{-1} \partial_n \rho_0. \qquad (11a,b)$$

The equation for the variation of gravitational potential is $\nabla^2 U_v = 4\pi\kappa_N \rho_v$, where $\kappa_N$ is Newton's gravitational constant. This equation can be transformed using Eqs. (5a,b), (7a,b) and (9a,b):

$$L_g^2 \nabla^2 U_v + U_v = \psi - \psi_w + u_n \left(v_P^2/g_1\right) N^2,$$

$$L_g^2 = R_1 \left(v_P^2/3g_1\right)\left(\rho_{Av}/\rho_0\right). \qquad (12a,b)$$

Here $g_1 = 10.68$ m/s$^2$ is the gravitational acceleration at the CMB, and $\rho_{Av} = 11.05$ g/cm$^3$ is the average density of the core. This gives approximately $L_g \approx (0.8 - 0.9) R_1$.

### 2.1. Boundary conditions

We assume the CMB is rigid. The earth's deformation produces significant and well known changes in the earth's moments of inertia, but the elastic displacements of the solid CMB are small, and can be neglected in the boundary conditions for the core fluid motion. The normal velocity component should be zero at the bottom of the ocean, $r_e = R_1$, and continuous at its surface, $r_e = R_S$:

$$v_n = 0 \ \ at\ r_e = R_1, \quad [[v_n]] = 0 \ \ at\ r_e = R_S. \quad (13a,b)$$

Here the pair of double square brackets denotes the jump in the quantity between them. Arbitrary jumps in the tangential velocities, $\mathbf{v}_\tau$, are admissible because we neglect viscosity and disregard the Ekman layers, which are very thin. On the solid CMB, $r_e = R_1$, an arbitrary pressure is admissible. The SOC surface, $r_e = R_S$, experiences an oscillating displacement $u_n$; here $\partial_t u_n = v_n(R_S)$. The total pressure on the displaced surface can be written in the linear approximation as $p_0 + u_n \partial_n p_0 + p_v$. Both the equilibrium pressure and the total pressure should be continuous; therefore the jump $[[\partial_n p_0]] = g_n [[\rho_0]]$ implies a jump of the pressure variation of $[[p_v]] = -u_n [[\partial_n p_0]]$. The pressure jump condition is therefore:

$$[[p_v]] = -u_n g_n [[\rho_0]] \quad at\ r_e = R_S. \qquad (14a)$$

The gravitational potential, $U_v$, should be regular at the center, $r = 0$, and continuous together with its gradient at $r_e = R_S$, and at $r_e = R_1$, to match the potential in the mantle.

Using $[[\psi]] = [[\rho_0^{-1} p_v]]$, and $[[\rho_0]] = -C_S \rho_0 (R_S + 0)$ one can transform Eq. (14a) into a jump condition for $\psi$:

$$[[\psi]] = C_S \left[u_n g_n + \rho_0^{-1} p_v (R_S - 0)\right] \quad at\ r_e = R_S. \qquad (14b)$$

## 3. Core fluid motion excited by the wobbling mantle

We start with the celebrated Poincaré solution which is valid in the simplest case of an incompressible fluid of constant density contained in a rotating rigid ellipsoidal container of ellipticity $\varepsilon$. This solution can be conveniently written down using the vectors of ''elliptical rotations'', $\mathbf{o}^x$ and $\mathbf{o}^y$, around the axis $\mathbf{1}_x$ and $\mathbf{1}_y$:

$$\mathbf{o}^x = \mathbf{1}_x \times \mathbf{r} - \varepsilon\nabla(yz), \quad \mathbf{o}^y = \mathbf{1}_y \times \mathbf{r} + \varepsilon\nabla(xz). \tag{15a,b}$$

The vectors $\mathbf{o}^j$ $(j = x, y)$ are tangential to the ellipsoids of ellipticity $\varepsilon$. They possess the property $\mathbf{1}_n \cdot \mathbf{o}^j = 0$, and $\mathbf{1}_z \times \mathbf{o}^j = \mathbf{1}_j z(1+\varepsilon)$, and for constant $\varepsilon$ they satisfy $\nabla \cdot \mathbf{o}^j = 0$, and $\nabla \times \mathbf{o}^j = 2\mathbf{1}_j$.

The Poincaré solution $(\mathbf{v}^\circ, \psi^\circ)$ of Eq. (8) with the inertial excitation term $\mathbf{f}^w$ on the right-hand side given by (10a), but without $\mathbf{f}^N$, can be written in the form:

$$\mathbf{v}^\circ = w_x \mathbf{o}^x + w_y \mathbf{o}^y = \mathbf{w} \times \mathbf{r} + \varepsilon\nabla(z\mathbf{s} \times \mathbf{w})_z, \tag{16}$$

$$\psi^\circ = -(2\Omega + \omega)(1+\varepsilon) z\mathbf{s} \cdot \mathbf{w}, \tag{17}$$

where

$$\mathbf{w} = w_x \mathbf{1}_x + w_y \mathbf{1}_y = -\omega \mathbf{m}/\delta_w, \quad \delta_w = 1 + \varepsilon + \sigma, \tag{18a,b}$$

and $\sigma = \omega/\Omega$. This solution satisfies the equation $\nabla \cdot \mathbf{v}^\circ = 0$ and the condition $v_n^\circ = 0$. The value Eq. (18a,b) of $\mathbf{w}$ is determined by the inertial excitation force $\mathbf{f}^w$. The Poincaré solution corresponds to $C_S = C_H = 0$, $\mathbf{f}^N = 0$, and $\nabla\varepsilon = 0$. Equations for the pressure, $p^\circ$, and the gravitational potential, $U^\circ$, are obtained from Eqs. (9a,b) and (12a,b), where $u_n{}^\circ = 0$ is substituted:

$$L_g^2 \nabla^2 U^\circ + U^\circ = \psi_w{}^\circ, \quad \rho_0^{-1} p^\circ = \psi_w{}^\circ - U^\circ, \tag{19a,b}$$

where

$$\psi_w{}^\circ = \psi^\circ - \psi_w = (\Omega/\sigma) z(\mathbf{s} \cdot \mathbf{w}) \Delta_w,$$

$$\Delta_w = 1 + \varepsilon(1 - \sigma - \sigma^2). \tag{20a,b}$$

To evaluate the rôle of the SOC we seek a solution as a sum of the Poincaré main motion, and an ''additional'' motion, which we distinguish by a dash. The total motion is described by $\mathbf{v} = \mathbf{v}^\circ + \mathbf{v}'$, $\psi = \psi^\circ + \psi'$, $U_v = U^\circ + U'$, and $p_v = p^\circ + p'$, where $(\mathbf{v}^\circ, \psi^\circ)$ are given by Eqs. (16), (17) and (18a,b) with $\varepsilon = \varepsilon(r_e)$. The property $v_n{}^\circ = 0$ and the boundary condition (13a) imply the simple boundary condition $v_n' = 0$ at $r_e = R_1$. We assume the additional motion to be a small perturbation of the main motion. Substituting $\mathbf{v} = \mathbf{v}^\circ + \mathbf{v}'$ and $\psi = \psi^\circ + \psi'$ in Eqs. (8), (9a,b), (10a,b), (11a,b), (12a,b) and (13a,b) with $\varepsilon$ depending on $r_e$, and $\nabla\varepsilon = \mathbf{1}_n \varepsilon_n'$ we obtain equations and boundary conditions for $\mathbf{v}'$ and $\psi'$. The right-hand sides of these equations contain the quantity $\varepsilon_n'$. We neglect the effects of ellipticity for the perturbation, and consider the solution for the perturbation in a spherical region, replacing $r_e$ by r, $\mathbf{1}_n$ by $\mathbf{1}_r$, and $\varepsilon_n'$ by $\varepsilon_r' = \partial_r \varepsilon$. We write $v_r{}^\circ = 0$ and $u_r{}^\circ = 0$ instead of $v_n{}^\circ = 0$ and $u_n{}^\circ = 0$, and utilize $v_r' = \partial_t u_r'$ in equations for the perturbation.

The gravitational potential, $U^\circ$, can be found from Eq. (19a,b), where the substitution, $U^\circ = \psi_w^\circ U_r^\circ(r)$, separates variables. After $U^\circ$ is found, one can obtain from Eq. (19a,b) the ratio $p^\circ/\rho_0 = \psi_w^\circ \pi^\circ(r)$, which is continuous; here $\pi^\circ(r) = 1 - U_r^\circ(r)$. The function $U_r^\circ(r)$ is smooth and rather small because $L_g^2 \nabla^2 U^\circ \sim 10U^\circ$. Neither $U_r^\circ(r)$ nor $\pi^\circ(r)$ changes much in the thin layer; we replace $\pi^\circ(r)$ in the SOC by a constant value $\pi^\circ(R_S)$, which is not far from the unity.

The equations for $\mathbf{v}'$ and $\psi'$ are:

$$\partial_t \mathbf{v}' + 2\Omega \mathbf{1}_z \times \mathbf{v}' + \nabla\psi' = \mathbf{f}', \quad \rho_0^{-1} p' = \psi' - U', \tag{21a,b}$$

$$\nabla \cdot \boldsymbol{v}' = k_\rho v_r' + \varepsilon_{div}',$$

$$\varepsilon_{div}' = -\varepsilon_r' 2(1-\varepsilon)\mu\mathbf{s} \cdot (\mathbf{w} \times \mathbf{1}_z). \tag{22a,b}$$

For the ABC, we have $\mathbf{f}' = \mathbf{f}^\varepsilon$, while for the SOC, $\mathbf{f}' = \mathbf{f}^\varepsilon + \mathbf{f}^N$, where:

$$\mathbf{f}^\varepsilon = \mathbf{1}_r \varepsilon_r' \Omega z(\mathbf{s} \cdot \mathbf{w})(2+\sigma)\sigma\delta_w^{-1}, \tag{23}$$

$$\mathbf{f}^N = \mathbf{1}_r (C_H/H)(\pi^\circ \psi_w{}^\circ - g_1 u_r'). \tag{24}$$

Here $\pi^\circ \approx 1 - U_r^\circ(R_S)$. The ratio $p^\circ/\rho_0$ is continuos but $p^\circ$ is not, because of the jump in $\rho_0$ at $r = R_S$. This produces a jump $[[\psi']]$ proportional to $[[\rho_0]] = -C_S\rho_0$, which is small and can be taken into account in the equations for the small perturbation. Eqs. (14b) and (19a,b) yield a condition at $r = R_S$:

$$[[\psi']] = C_S[\pi^\circ \psi_w{}^\circ(R_S) - g_1 u_r'(R_S)]. \tag{25}$$

The small primed value, $u'_r$, is retained in the terms (24) and (25) because $u'_r$ is multiplied there by a large coefficient, $g_1 \sim \Omega^2 R_S/\varepsilon$.

Eqs. (12a,b), (19a,b) and (20a,b) give an equation for the gravity perturbation:

$$L_g^2 \nabla^2 U' + U' = \psi' + u'_r \left( v_P^2/g_1 \right) N^2. \quad (26)$$

Equations and boundary conditions for the additional motion (21)–(25) does not contain $U'$, and can be solved without knowing this quantity. The gravity perturbation, $U'$, can be calculated from Eq. (26) after $\psi'$ and $u'_r$ are found.

## 4. The motion induced by the SOC

The motion ($\mathbf{v}'$, $\psi'$) is generated by the force, $\mathbf{f}^{\varepsilon} \sim \varepsilon'_r$, due to the gradient of ellipticity, and by the forces due to $\mathbf{f}^N \sim C_H$, and due to the density jump, $C_S$. The effect of the ellipticity gradient is very small, $\varepsilon'_r \sim 10^{-1}\varepsilon/R_1 \sim 3\times10^{-4}/R_1$, and it is not generated by the SOC. We will not consider this effect. The SOC-effects we are interested in are proportional to the small quantities $C_S \sim 10^{-3}$ and $C_H \sim 10^{-4}$; thus, we expect that $v' \ll v^{\circ}$.

In finding the perturbation due to effects of the SOC we exploit two strong simplifications. First, we replace the continuity Eq. (22a,b) by a simpler one, $\nabla \cdot \mathbf{v}' = 0$, which corresponds to a model of two Boussinesq fluids of constant density with $C_S = (\rho_{ABC} - \rho_{SOC})/\rho_{SOC}$. Second, we exploit the property $H/R_1 \sim 1/40 \ll 1$, and consider only the leading terms in $H/R_1$. The latter approximation makes it possible to separate the solutions in the SOC and ABC, because it gives $\psi'_{ABC} \sim (H/R_1)\psi'_{SOC} \ll \psi'_{SOC}$. Therefore one can neglect the quantity $\psi'_{ABC}$ as compared with $\psi'_{SOC}$ in the jump condition for $[[\psi']]$. This can be shown by the following estimates: in the SOC we have $v'_r \sim (H/R_1)v'_\tau \ll v'_\tau$, but in the ABC $v'_r \sim v'_\tau$, while in both regions $\psi' \sim (\Omega R_1)v'_\tau$; it follows from $v_r{}'_{ABC} \sim v_r{}'_{SOC}$ that $v'_{\tau ABC} \sim (H/R_1)v'_{\tau SOC}$, and we obtain the desired relation, $\psi'_{ABC} \sim (H/R_1)\psi'_{SOC}$. We assume approximately $[[\psi']] \approx \psi'_{SOC} = \psi'(R_S + 0)$, thus turning Eq. (25) into a boundary condition for the SOC. Hence, the solution for the SOC can be found independently from that in the ABC. After this is done one can find a solution for the ABC using the boundary condition $v'_{r\,ABC}(R_S) = v'_{r\,SOC}(R_S)$.

We use the spherical coordinates, and write $\mathbf{v}' = \mathbf{1}_r v'_r + \mathbf{v}'_\tau$, $\mathbf{v}'_\tau = \mathbf{1}_\theta \mathbf{v}'_\theta + \mathbf{1}_\phi \mathbf{v}'_\phi$, $\nabla_\tau \psi' = r^{-1}(\mathbf{1}_\theta \partial_\theta \psi' + \mathbf{1}_\phi \mu_s^{-1} \partial_\phi \psi')$, $\mu = \cos\theta$, $\mu_s = \sin\theta$. Equations and boundary conditions for the SOC (with $\mathbf{f}^{\varepsilon} = 0$) take the form:

$$\partial_t \mathbf{v}'_\tau + 2\Omega\mu \mathbf{1}_r \times \mathbf{v}'_\tau + \nabla_\tau \psi' = 0, \quad (27)$$

$$\left\{ \partial_t v'_r - 2\Omega\mu_s v'_\phi \right\} + \partial_r \psi' = f_r^N = (C_H/H)\left[ (\Omega/\sigma)\pi^{\circ} R_S^2\, \mu\mu_s w_s - g_1 u'_r \right], \quad (28)$$

$$\partial_r v'_r = -\nabla_\tau \cdot \boldsymbol{v}'_\tau, \quad v'_r(R_1,\theta,\phi) = 0, \quad (29a,b)$$

$$\psi'(R_S,\theta,\phi) = C_S\left[ (\Omega/\sigma)\pi^{\circ} R_S^2\, \mu\mu_s w_s - g_1 u'_r \right] \text{ at } r = R_S. \quad (30)$$

Here we use $z \approx R_S\,\mu$, $\mathbf{s} \approx R_S\,\mu_s \mathbf{1}_s$ in the SOC, thus $z(\mathbf{s}\cdot\mathbf{w}) \approx R_S^2\,\mu\mu_s w_s$. We write also $\partial_r v'_r$ instead of $r^{-2}\partial_r(r^2 v'_r)$ in Eq. (29a,b), and we replace $\Delta_w$ by 1 in Eq. (20a,b) omitting terms $\sim \varepsilon$. Exploiting $v'_r/v'_\tau \approx H/R_S \ll 1$ we neglect the term $2\Omega\mu_s v'_r$ in Eq. (27) — this approximation is called ''traditional'' in the ocean sciences. The estimates show that the terms in curly brackets in (28) are small, $\sim H/R_S$, and we neglect them. Thus Eq. (28) becomes simply $\partial_r \psi' = f_r^N$.

Let us consider the Chandler wobble; then $\sigma \sim \varepsilon \ll 1$, and assuming $\sigma = 0$ in Eq. (27), we can easily express $\mathbf{v}'_\tau$ through $\psi'$. We combine Eq. (29a,b), Eq. (30) with Eqs. (27) and (28) in the form:

$$\mathbf{v}'_\tau = (2\Omega\mu)^{-1} \mathbf{1}_r \times \nabla_\tau \psi',$$

$$\partial_r \psi' = (C_H/H)\left[ (\Omega/\sigma)\pi^{\circ} R_S^2\, \mu\mu_s w_s - g_1 u'_r \right], \quad (31a,b)$$

and get $\nabla_\tau \cdot \mathbf{v}'_\tau = -(2\Omega R_S\,\mu_s\,\mu^2)^{-1}\partial_\phi \psi'$ from Eq. (31a,b). Using relations $\partial_\phi w_s = w_\phi$, $\partial_\phi w_\phi = -w_s$, and $v'_r = \partial_t u'_r = -\Omega\sigma\partial_\phi u'_r$, we obtain the following equation and boundary conditions for $v'_r$:

$$H^2 \partial_r^2 v'_r - q^2 v'_r = q^2 \mu_s\, \mu v_r^I, \quad (32)$$

$$v'_r = 0 \text{ at } r = R_1,$$

$$(C_H/C_S)\, H \partial_r v'_r - q^2 v'_r = q^2 \mu_s\, \mu v_r^I, \text{ at } r = R_S, \quad (33a,b)$$

where $q = \kappa_g/\mu$, and

$$\kappa_g^2 = (g_1 H C_H / 2\sigma)(\Omega R_S)^{-2},$$

$$v_r^I = \pi^\circ \Omega^2 R_S^2 g_1^{-1} w_\phi = \pi^\circ \varepsilon_g R_S w_\phi. \quad (34a,b)$$

Here $\varepsilon_g = \Omega^2 R_S g_1^{-1} \approx 1/600$. For $C_H = 10^{-4}$, $\sigma = 1/435$ the expression (34a) gives $\kappa_g = 0.55$, hence $q \sim 1$ if $\mu$ is not very small. Note that $v_r^I$ is not a constant; it is proportional to $w_\phi(t, \phi)$.

Integration of Eq. (32) over the radius with the boundary conditions (33a,b) yields:

$$v_r' = -v_r^I \mu_s \mu Y(qx), \quad \boldsymbol{x} = (R_1 - r)/H, \quad (35a,b)$$

where the non-dimensional radial coordinate (35b) is introduced, and it is denoted:

$$Y = 1 - \cosh(qx) + a_q \sinh(qx),$$

$$a_q = [q + \gamma \tanh(q)][\gamma + q\tanh(q)]^{-1}. \quad (36a,b)$$

Here $\gamma = C_H/C_S$ for brevity. If $\gamma = 0.1$ then for $q = 0.5$, 1, and 2 we have $a_q = 1.65$, 1.25, and 1.034; if $\gamma = 1$ then for $q = 0.5$, 1, and 2 we have $a_q = 0.78$, 1.00, and 1.012. In these cases, we have $Y \sim 1$, and $v_r' \sim v_r^I$. For $q \ll 1$ we have $a_q \approx q(1+\gamma)/(\gamma + q^2)$; if $q^2 \ll \gamma$ then $a_q \approx q(1+\gamma^{-1})$. For small $\mu$, that is, for $q \gg 1$, we have $\tanh(q) = 1$ hence $a_q = 1$, and $\cosh(qx) \approx \sinh(qx)$. The hyperbolic functions cancel out in $Y(qx)$, and the velocity is $v_r' = -v_r^I \mu_s \mu$.

Substituting Eqs. (31a,b) and (29a,b), and using $\partial_\phi^2 \psi' = -\psi'$, we express $\psi'$ through $\partial_r v_r'$:

$$\psi' = (\pi^\circ C_H/\sigma)\Omega R_S^2 \mu \mu_s w_s Y^\dagger,$$

$$Y^\dagger = q^{-1}\left[a_q \cosh(qx) - \sinh(qx)\right]. \quad (37a,b)$$

Here $Y^\dagger \sim 1$ typically, but $Y^\dagger \sim \gamma^{-1} = C_S/C_H$ for $q \ll 1$ and $\gamma \ll 1$. A straightforward calculation gives:

$$v_\theta' = -v_\theta{}^\circ(\pi^\circ C_H/2\sigma)Y^\dagger,$$

$$v_\phi' = -v_\phi{}^\circ(\pi^\circ C_H/2\sigma)\mu^{-2}\partial_\theta(\mu_s \mu Y^\dagger). \quad (38a,b)$$

Here, the approximation $\mathbf{v}_\tau{}^\circ \approx R_S(\mathbf{w} \times \mathbf{1}_r)$ is used for the SOC. Expressions (38a,b) give the ratio of the additional velocity in the SOC to the main velocity of order of $v_\tau'/v_\tau{}^\circ \sim C_H/2\sigma$. For $C_H \sim 10^{-4}$, we have $C_H/2\sigma \sim 0.02$. Hence the perturbation approach is valid.

The solution in the ABC is governed by Eqs. (27), (28) and (29a,b) with $f_r^N = 0$. It can be found as an expansion in the small parameter $\sigma$. In the zeroth approximation this solution, $\mathbf{v}^{(0)}$, is quasi-stationary, and satisfies the Proudman–Taylor's theorem. The velocity $\mathbf{v}^{(0)}$ has only a $z$-component, $\mathbf{v}^{(0)} = \mathbf{1}_z v_z^{(0)}(s, \phi)$. Its radial projection, $v_{rABC}^{(0)} = \mu v_z^{(0)}$, has to match $v_{r\,SOC}'$ at $r = R_S$. At this radius $x = 1$, $\mu_s = s/R_S$ and $\mu = (R_S^2 - s^2)^{1/2}/R_S$. Thus we have

$$v_z^{(0)}(s,\phi) = -v_r^I(s/R_S)Y(q_S),$$

$$q_S = \kappa_g R_S/\left(R_S^2 - s^2\right)^{1/2}. \quad (39a,b)$$

The first approximation quantities, $v_s^{(1)} = \sigma(z/2s) v_z^{(0)}$, $v_\phi^{(1)} = \sigma(z/2)\partial_s \partial_\phi v_z^{(0)} = \partial_\phi \partial_s (s v_s^{(1)})$, and $\psi^{(1)} = \sigma \Omega z \partial_\phi v_z^{(0)}$, are negligibly small being proportional to $\sigma \sim \varepsilon$.

It should be noted that the effect of the SOC on the CW is greatly enhanced by the presence of a small factor $\sigma \sim \varepsilon$ in the denominator of Eq. (38a,b). Therefore the effect is much greater than the very small value of the density excess, $C_H$, in the SOC. The core fluid motion decreases the period of the CW by approximately 35 days, see Smith and Dahlen (1981). The additional motion due to the SOC is mostly concentrated in the SOC, which has a smaller moment of inertia than that of the core, their ratio being $5H/R_1 \approx 1/8$. Its velocity is smaller in size than that of the main motion by approximately the factor $C_H/2\sigma \sim 0.02$ and has the opposite sign; see Eq. (38a,b). Hence it increases the period of the CW by approximately $35 \times 0.02/8 \sim 0.1$ day. This is much smaller than the probable error, $\sim 3$ day, in the observed value of $T_{CW} = 435$ days.

Tangential components of the additional velocity are much greater in the SOC than in the ABC, and experience a jump, $[[v_\tau]] \sim v_\tau' \sim (C_H/2\sigma)v_\tau{}^\circ$, at the surface of the SOC. This velocity discontinuity ''cuts'' the geomagnetic field lines inside the highly conducting core, thus generating electric current and Joule dissipation, $Q_J^S$, which contributes to the decay of the CW. Braginsky (1999) compared the effect of this ''cutting'' at the SOC surface with the Joule dissipation in the mantle, $Q_J^M$, due to mantle conductivity, and obtained $Q_J^S/Q_J^M \approx (\eta_M/2\eta)^{1/2}([[v_\tau]]/v_M)^2$ — a nearly obvious estimate. Here $v_M$ is the fluid velocity on the CMB; $\eta$ and $\eta_M$ are the magnetic diffusivities of the core and the lower mantle, which are inversely proportional to their conductivities; hence $\eta_M/\eta \gg 1$. We assume $\eta = 2$ m $s^{-2}$ following Braginsky and Roberts (1995). The CW decay time due to the Joule dissipation, $\tau_D$, is inversely proportional to $Q_J$.

To assess the influence of the SOC on the CW decay, it is convenient to use the results of Buffett (1992) who considered the CW decay due to $Q_J^M$ in detail. He considered two variants of the mantle conductivity: his ''Profile A'' corresponds to a homogeneous mantle of conductivity $10^3$ S $m^{-1}$ (that is $\eta_M = 8 \times 10^2$ m $s^{-2}$) and gives $\tau_{DA} = 1.57 \times 10^5$ years; his ''Profile B'' includes a hypothetical layer in the lower mantle, of the same conductivity as the core, which enhances the dissipation, and this gives $\tau_{DB} = 1.35 \times 10^4$ years. Using $\eta_M$ for Profile A, we have $(\eta_M/2\eta)^{1/2} = 14$, and with $[[v_\tau]]/v_M \sim 0.02$ we obtain $\tau_D \sim \tau_{DA}(14 \times 0.02^2)^{-1} \sim 3 \times 10^7$ years. It is interesting to note that the skin layer inside the core, where the magnetic field lines are ''cut'', plays the same rôle as the specific layer of Profile B that has the same conductivity as the core. If it were $[[v_\tau]] \sim v_M$, then we would get a decay time, $\tau_D \sim \tau_{DA}/14 \sim 10^4$ years, similar to that of Profile B. All these estimated decay times are much longer than the observed $\sim$30-year decay time, which is attributed to mantle anelasticity. We conclude that the SOC effect on the decay of the CW is negligibly small despite the enhancements due to the factor $\sigma^{-1} \sim \varepsilon^{-1}$, and due to the magnetic line cutting.

The effect of the SOC on gravity variations also is small. The main gravity variation due to the Chandler wobble can be estimated as $\Delta g \sim \Omega^2 |\mathbf{m}| R_E \sim 2$ $\mu$gal, where $R_E$ is the radius of the earth; this effect was observed with superconducting gravimeters, see, e.g., Hinderer and Legros (1989). It is possible to single out the gravity perturbation originating from the wavelike Poincaré motion in the fluid core ($\mathbf{v}^\circ$, $\psi^\circ$) utilizing its characteristic space-time dependence. Using Eqs. (19a,b) and (20a,b) we estimate that $\Delta g^\circ(R_1)$ is of order $\Delta g^\circ(R_1) \sim 10^{-1}\Omega^2|\mathbf{m}|R_1$; hence, $\Delta g^\circ = \Delta g^\circ(R_E) \sim 10^{-1}\Omega^2|\mathbf{m}|R_1^3/R_E^2 \sim 10^{-1}(R_1/R_E)^3 \Delta g \sim 30$ ngal — a small quantity which is difficult to observe. Assume in analogy with Eq. (38a,b) that the effect of the SOC on the gravity variation is $\Delta g' \sim (C_H/2\sigma)\Delta g^\circ \sim 2 \times 10^{-2}$ $\Delta g^\circ$; we then obtain the tiny quantity $\Delta g' \sim 1$ ngal.

All the above calculations of the effects due to the SOC are very rough. They give only order of magnitude estimates, but the complications of an accurate calculation of such small effects would not be justified.

Another interesting variation in the earth's orientation is that of nutation. In this case $\sigma = -1 + \Delta\sigma$, where $\Delta\sigma \ll 1$, and so the enhancement $\sim \varepsilon^{-1}$ is absent. Therefore one can expect that the effect of the SOC on nutation is weaker than its effect on the CW. An order of magnitude analysis shows that $v'_\tau \sim C_S v_\tau{}^\circ$; thus the effect of the SOC on nutation is a very small indeed.

## 5. Conclusion

The influence of the SOC on the motions in Earth's fluid core excited by the Chandler wobble is considered. The fluid velocity is assumed in the form $\mathbf{v}^\circ + \mathbf{v}'$, where the main motion, $\mathbf{v}^\circ$, arises in the absence of the SOC, and $\mathbf{v}'$ is a small perturbation due to the SOC. An approximate solution is found for $\mathbf{v}'$, which is of order $v' \sim (C_H/2\sigma)v^\circ$. The small denominator entering the coefficient in this estimate, $1/2\sigma \sim 200$, greatly enhances the magnitude of the perturbation velocity. The origin of this enhancement factor can be traced to the expression for the Archimedean buoyancy force, which is proportional to the fluid displacement, $u \sim v/\omega$. Despite this enhancement the influence of the SOC with $C_H \sim 10^{-4}$ upon the CW is very small. The SOC increases the CW period, $T_{CW}$, by about 0.1 day, which is less than the error in observation of $T_{CW}$. An additional energy dissipation due to the perturbation velocity is about six orders of magnitude smaller than the one that explains the decay of the CW. The influence of the SOC on nutation is even much smaller, the added velocity being of order only $C_S v^\circ$.

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