Phase Changes and Mantle Convection

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In a two-phase fluid layer heated from below, two instability mechanisms are present, the ordinary Rayleigh instability associated with thermal expansion and a phase change instability driven by the density difference between the phases. A stability analysis is presented for the combined effects of these instability mechanisms, and the critical Rayleigh number is determined as a function of the properties of the phase transition. The results are applied to the olivine-spinel phase change in the mantle, and it is concluded that this phase change in the presence of a negative temperature gradient may intensify deep mantle convection. The elevation of the phase-change boundary within the descending cold lithospheric plate at ocean trenches is a finite amplitude example of the phase-change instability. The additional gravitational body force on the slab due to the elevation of the phase boundary is comparable to that of thermal contraction.

The concept of plate tectonics is now well accepted. By postulating relative movement between segments of the rigid, spherical outer shell of the earth, many geological phenomena can be explained. A problem of critical importance is the nature of the driving mechanism for plate tectonics. This driving mechanism must provide the energy dissipated in earthquakes and volcanism. The only steady-state source of energy of sufficient magnitude appears to be the heat generated by the radioactive elements in the earth's mantle. Other possible energy sources are the heat generated by the formation of the earth's core or other continuing differentiation processes.

Heating can result in large-scale motions only if a body behaves like a fluid. There is ample evidence from uplift phenomena that over long periods of time the earth's mantle does exhibit fluidlike behavior. Physical mechanisms for this behavior of a crystalline solid are also available [*Gordon,* 1965; *Weertman,* 1970].

If a fluid with a positive coefficient of thermal expansion is heated from below, thermal convection will occur if the Rayleigh number exceeds a critical value. Since it was first proposed by *Holmes* [1931] as an explanation for continental drift, this type of thermal convection has been the favored mechanism for driving large-scale surface displacements. A number of authors [e.g. *Pekeris,* 1935; *Knopoff,* 1964] have shown that the Rayleigh number for the upper mantle exceeds the critical value and others [e.g. *Turcotte and Oxburgh,* 1967; *Oxburgh and Turcotte,* 1968; *McKenzie,* 1969] have discussed the structure of convection cells in the mantle.

Evidence from both seismology [*Johnson,* 1967] and geochemistry [*Ringwood,* 1970] indicates that an olivine–spinel phase change occurs at a mean depth near 400 km. The density change across this phase transition is about 7%. For comparison, the density change associated with a temperature difference of 1000°K is 3%. Since density differences drive mantle convection it is expected that the olivine–spinel phase change will have an important effect. Seismic evidence also indicates another important solid–solid phase change at a mean depth of 650 km. Also, significant density differences would be associated with partial melting in the mantle.

For the olivine–spinel phase change the dense phase (spinel) lies beneath the light phase (olivine), and heat is evolved when going from olivine to spinel. If this phase transition is described by a univariant system, then in thermodynamic equilibrium the phase-change boundary must lie on the Clapeyron curve. The slope of this curve dp/dT is positive, and the coefficients of thermal expansion for both phases are also positive. Under isothermal conditions such a phase boundary is stable. Consider an element of fluid that moves downward through the phase-change boundary. When the fluid passes through the boundary, heat is evolved, the fluid element is heated, its density decreases because of thermal expansion, and there is an upward stabilizing body force. A second stabilizing effect is due to the displacement of the phase boundary. Since the phase boundary must lie on the Clapeyron curve, assuming thermodynamic equilibrium, the heat evolved when the fluid element passes through the phase-change boundary will cause the boundary to be displaced to a greater depth (larger hydrostatic pressure). With the boundary displaced downward, the lighter fluid above the phase boundary will experience an upward, stabilizing hydrostatic head relative to an undisturbed column of fluid.

Knopoff [1964] has approximated the phase-change boundary as a region of nonadiabatic density distribution and concluded that the phase boundary acts as a 'positive barrier to convection.' *Verhoogen* [1965] considered the penetration of an element of fluid through a phase-change boundary and concluded that the phase boundary acts as an 'obstacle to convection.' *Vening Meinesz* [1962] carried out a partial analysis of a continuous phase change in a descending element of fluid and concluded that the phase change could contribute to instability.

Schubert et al. [1970] have shown that the olivine–spinel phase change may be destabilizing in the presence of a negative temperature gradient. Again, consider a fluid element that moves downward through the phase change. Because of the zero-order temperature gradient, the fluid element approaching the phase boundary will be cooler than undisturbed fluid at the phase boundary. Assuming thermodynamic equilibrium so that the phase boundary lies on the Clapeyron curve, the phase boundary will be displaced upward to a lower hydrostatic pressure. With the phase-change boundary displaced upward, the heavier material below the boundary provides a hydrostatic pressure head tending to drive the flow downward, leading to instability. This instability acts in addition to the Rayleigh instability. However, as discussed above, the downward flow of fluid through the boundary releases heat, thus tending to warm the fluid and return the phase boundary to its unperturbed location. The negative temperature gradient promotes instability, whereas the release of heat in the phase change promotes stability.

The effect of this phase change on mantle convection can be seen in discussing the descending slab of cold lithosphere beneath an oceanic trench. Since this descending slab is cold, the olivine–spinel phase change will occur at a shallower depth in the slab than in the adjacent mantle. The high-density spinel phase will give an additional body force on the slab that will help to pull it down into the mantle.

The phase-change instability was first discussed by *Busse and Schubert* [1971]. In the present paper additional results are presented for the combined effects of the Rayleigh instability and the phase-change instability. The results are applied to the olivine–spinel phase change in the mantle.

THEORETICAL MODEL

Consider the stability against infinitesimal perturbations of a static two-phase fluid layer confined between the horizontal planes $z = \pm d$ with a horizontal univariant phase boundary separating the two phases at $z = 0$. The fluid is assumed to be in thermodynamic equilibrium; the location of the phase boundary is thus determined by the intersection of the Clapeyron curve with the pressure-temperature curve for the fluid layer. In the perturbed state the phase boundary will be distorted from its initial position at $z = 0$. The slope of the Clapeyron curve is given by

$$\gamma \equiv \left(\frac{dp}{dT}\right)_c = \frac{Q\rho_1\rho_2}{T\Delta\rho} \qquad (1)$$

where p is the pressure, T is the absolute temperature, $\Delta\rho$ is the change in density at the phase transition ρ_2–ρ_1 (the subscripts 1 and 2 refer to the upper and lower phases, respectively), and Q is the energy per unit mass required to change material of phase 2 into material of phase 1. Thus, if the less dense phase lies above the more dense phase, the slope of the pressure-temperature curve exceeds the slope of the Clapeyron curve, and Q and $\Delta\rho$ are both positive.

We assume that $\Delta\rho \ll \rho_1, \rho_2$ and that both phases have the same values of absolute viscosity μ, thermal conductivity k, specific heat at constant pressure c_p, thermal diffusivity κ, and kinematic viscosity ν (μ, k, c_p, κ, and ν are constants). Each phase is assumed to be a Boussinesq fluid, i.e., the density of each phase is regarded as constant except insofar as the thermal expansion of the fluid provides a force of buoyancy. The value of the coefficient of thermal expansion α is assumed to be constant and the same for both phases. The difference in density between the two phases is taken into account in determining the distortion of the phase-change boundary and in the pressure-boundary condition at the phase-change interface. Two instability mechanisms are present in this model, the ordinary Rayleigh instability associated with the thermal expansion of the fluid and a phase-change instability driven by the density difference between the phases.

In the undisturbed state there are a constant negative temperature gradient of absolute magnitude β throughout the fluid layer and pressure gradients $-\rho_1 g$ and $-\rho_2 g$ in the upper and lower phases, respectively (g is the acceleration of gravity). The static state is, of course, horizontally homogeneous. As noted previously, if less dense fluid is above more dense fluid $\rho_{1,2}\, g/\beta > \gamma$.

The linearized equations for the velocity perturbations $\mathbf{u}_{1,2}$, the temperature perturbations $\theta_{1,2}$ and the pressure perturbations $\pi_{1,2}$ are

$$\nabla \cdot \mathbf{u}_{1,2} = 0 \qquad (2)$$

$$-\frac{1}{\rho} \nabla \pi_{1,2} + \nu \nabla^2 \mathbf{u}_{1,2} + g\alpha\theta_{1,2}\hat{\mathbf{z}} = 0 \qquad (3)$$

$$-(\beta - \beta_a) w_{1,2} = \kappa \nabla^2 \theta_{1,2} \qquad (4)$$

where $\rho \approx \rho_1 \approx \rho_2$, w is the vertical component of the perturbation velocity, $\hat{\mathbf{z}}$ is a unit vector in the positive z direction, and β_a is the adiabatic temperature gradient given by

$$\beta_a = \left(\frac{\partial \rho^{-1}}{\partial T}\right)_p \frac{g\rho T}{c_p} \qquad (5)$$

The adiabatic temperature gradient is assumed to have the same value for both phases. The right-hand side of equation 5 is to be evaluated at the position of the undisturbed phase boundary. In writing the momentum and energy conservation equations, equations 3 and 4, respectively, we have assumed that the state of marginal stability is given by $\partial/\partial t = 0$, i.e., that the principle of exchange of stabilities is valid.

Without loss of generality we consider the two-dimensional periodic solutions of equations 2–4 with horizontal wave number l. The vertical velocity perturbations are solutions of

$$\left(\frac{\partial^2}{\partial z^2} - l^2\right)^3 w_{1,2} = \frac{-(\beta - \beta_a)\alpha g l^2}{\kappa\nu} w_{1,2} \qquad (6)$$

The horizontal boundaries at $z = \pm d$ are taken to be free surfaces. Thus we require

$$w = \frac{\partial^2 w}{\partial z^2} = 0 \quad \text{at} \quad z = \pm d \qquad (7)$$

The symmetric solutions of equation 6 subject to conditions of equation 7 are

$$w_{1,2} = \sum_{n=1}^{3} A_n \sinh \delta_n\left(1 \pm \frac{z}{d}\right) \qquad (8)$$

where

$$\delta_n^2 = l^2 d^2 + (R_\beta l^2 d^2)^{1/3} e^{i(\pi/3)(2n-1)}$$

$$n = 1, 2, 3 \qquad (9)$$

$$R_\beta = \frac{\alpha(\beta - \beta_a) g d^4}{\kappa\nu} \qquad (10)$$

A_n are constants of integration, and the upper and lower signs on the right-hand side of equation 8 refer to regions 2 and 1, respectively.

With the solution for the vertical velocity perturbation (8), we can determine the temperature perturbation from (4) and the boundary conditions

$$\theta = 0 \quad \text{at} \quad z = \pm d \qquad (11)$$

The symmetric solutions for the temperature perturbations are

$$\theta_{1,2} = \sum_{n=1}^{3} A_n \frac{\nu(\delta_n^2 - l^2 d^2)^2}{\alpha g d^2 (l^2 d^2)} \sinh \delta_n\left(1 \pm \frac{z}{d}\right) \qquad (12)$$

where the upper and lower signs refer to regions 2 and 1, respectively.

At the phase boundary there must be continuity of mass flow across the boundary, temperature and tangential stress. In our linear analysis these continuity conditions may be applied at $z = 0$. Thus

$$w_1 = w_2 \qquad \theta_1 = \theta_2$$
$$\frac{\partial^2 w_1}{\partial z^2} = \frac{\partial^2 w_2}{\partial z^2} \quad \text{at} \quad z = 0 \tag{13}$$

The solutions (8) and (12) satisfy conditions (13).

There are three remaining conditions at the phase boundary. The tangential velocity must be continuous at the interface between the phases. The linearization of this condition yields

$$\frac{\partial w_1}{\partial z} = \frac{\partial w_2}{\partial z} \quad \text{at} \quad z = 0 \tag{14}$$

As a result of the mass flux across the phase-boundary, energy will be released or absorbed depending on the direction of the phase change. At the phase boundary an amount of energy $\rho w Q$ is absorbed per unit time and per unit area. The energy absorbed or released at the interface must be balanced by the difference in the perturbation heat flux into and out of the phase boundary. This linearized condition is

$$\rho w Q = k\left(\frac{\partial \theta_1}{\partial z} - \frac{\partial \theta_2}{\partial z}\right) \quad \text{at} \quad z = 0 \tag{15}$$

Finally, the normal stress must be continuous at the phase boundary. This condition may be written

$$\pi_1 - \pi_2 = -g\eta\Delta\rho \quad \text{at} \quad z = 0 \tag{16}$$

where η is the vertical displacement of the distorted phase boundary. In condition (16) the difference in perturbation pressure between the two phases is equated to the hydrostatic head generated by the density difference between the phases and the displacement of the phase boundary. This pressure difference forces the flow that can result in a phase change driven instability.

The boundary between the phases must lie on the Clapeyron curve; η can thus be related to the temperature perturbations and pressure perturbations by

$$\eta = \frac{\pi_1 - \gamma\theta_1}{g\rho_1 - \gamma\beta} = \frac{\pi_2 - \gamma\theta_2}{g\rho_2 - \gamma\beta} \tag{17}$$

where the temperature and pressure perturbations in (17) are evaluated at $z = 0$, since the displacement of the phase boundary is infinitesimal. Equation 17 is easily understood when written in the form

$$\gamma = \left(\frac{\pi_1 - g\rho_1\eta}{\theta_1 - \beta\eta}\right)_{z=0} = \left(\frac{\pi_2 - g\rho_2\eta}{\theta_2 - \beta\eta}\right)_{z=0} \tag{18}$$

The numerators are the differences in pressure between a point located at $z = \eta$ in the perturbed state and a point located at $z = 0$ in the unperturbed state. The denominators are similar temperature differences.

DISCUSSION OF THE SOLUTION

When the solutions, equations 8 and 12, are substituted into the interface conditions, equations 14–16, a set of algebraic equations for the constants A_n ($n = 1, 2, 3$) is obtained. For this set to be solvable the determinent of the coefficients must be zero. This provides an eigenvalue equation for the problem as follows

$$0 = \begin{vmatrix} \delta_1 & \delta_2 & \delta_3 \\ S(\delta_1^2 - L^2)^2 \tanh \delta_1 + 2\delta_1^3 & S(\delta_2^2 - L^2)^2 \tanh \delta_2 + 2\delta_2^3 & S(\delta_3^2 - L^2)^2 \tanh \delta_3 + 2\delta_3^3 \\ \frac{2\delta_1}{L^2}(\delta_1^2 - L^2)^2 + R_Q \tanh \delta_1 & \frac{2\delta_2}{L^2}(\delta_2^2 - L^2)^2 + R_Q \tanh \delta_2 & \frac{2\delta_3}{L^2}(\delta_3^2 - L^2)^2 + R_Q \tanh \delta_3 \end{vmatrix} \tag{19}$$

where

$$S = \frac{\Delta\rho/\rho}{\alpha d\left(\frac{\rho g}{\gamma} - \beta\right)} \tag{20}$$

$$R_Q = \frac{\alpha g d^3 Q/c_p}{\nu\kappa} \tag{21}$$

$$L = ld \tag{22}$$

The parameter R_β, the ordinary Rayleigh number, enters equation 19 through the δ_n. The parameter S is the ratio of the fractional density change in the phase transition to the fractional density change associated with thermal expansion. Since $\rho g/\gamma > \beta$ and $\Delta\rho > 0$ when the heavy phase lies below the light phase, S is a positive quantity. Since S measures the magnitude of the fractional density change in the phase transition relative to that associated with thermal expansion, it is clear that

$$R_\beta S = \frac{gd^3\Delta\rho/\rho}{\kappa\nu} \frac{(1 - \beta/\beta_a)}{\left(\frac{\rho g}{\gamma\beta} - 1\right)} \qquad (23)$$

is the appropriate Rayleigh number for the phase-change density difference. Just as the ordinary Rayleigh number R_β measures the effectiveness of density differences due to thermal expansion in forcing convective instability, so the phase change Rayleigh number $R_\beta S$ measures the effectiveness of phase change density differences in driving instability. The parameter R_Q is still another Rayleigh number based on the temperature difference Q/c_p instead of the destabilizing temperature difference across the fluid layer. This parameter measures the stabilizing influence of the latent heat in the phase transition.

The limiting cases $\alpha \to 0$ and $\beta \to \beta_a$, $\alpha \to 0$ and $\beta \neq \beta_a$, $\alpha \neq 0$ and $\beta \to \beta_a$ have been studied in detail by *Busse and Schubert* [1971]. The results of the analysis for the case $\alpha \to 0$, $\beta \neq \beta_a$ have been applied to mantle phase changes by *Schubert et al.* [1970]. This case is of particular interest since for $\alpha = 0$ the ordinary Rayleigh instability is not present, and one may focus on the phase change as the source of instability. As previously discussed, the inflow of relatively cold material from above the phase-change boundary (owing to the zero-order temperature gradient) forces the interface to a region of lower hydrostatic pressure, i.e., upward. With the interface displaced upward, the heavier material below the interface gives a hydrostatic pressure head tending to drive the flow downward, leading to instability. However, the downward flow of fluid through the interface releases heat, thus tending to warm the fluid and return the phase boundary to its unperturbed location. The inflow of cold material tends to promote instability, whereas release of heat by the phase change promotes stability. The ratio R_β/R_Q provides a quantitative measure of the opposing effects. It is important to note that in regions of downward flow the phase boundary is displaced upward and in regions of upward flow the phase boundary is moved downward.

In the general case, the Rayleigh number R_β is given by equation 19 as a function of the dimensionless horizontal wave number L and the parameters S and R_Q. For given values of S and R_Q, a particular value of L will yield a minimum value of R_β. Values of this minimum critical Rayleigh number $R_{\beta_{crit}}$ for the symmetric case are given in Figure 1 as a function of R_Q for various values of the parameter S. For $R_Q = S = 0$ there is no phase change, and we recover the case of ordinary Rayleigh instability for which $R_{\beta_{crit}} = 27\pi^4/2^6 = 41.094$ [*Chandrasekhar*, 1961]. In the limit $\alpha \to 0$, R_β and $R_Q \to 0$, $S \to \infty$ and $R_\beta S \to 40.923$ for $R_Q/R_\beta = 0$ [*Busse and Schubert*, 1971; *Schubert et al.*, 1970]. In the ordinary Rayleigh instability, the density change due to thermal expansion is spread throughout the fluid layer. In the phase change instability of a fluid with $\alpha = 0$, the density change occurs at a single position in the fluid. The Rayleigh number for the former problem can be found analytically, whereas for the latter the determination of the Rayleigh number requires the numerical evaluation of a transcendental expression. It is remarkable that the two Rayleigh numbers differ by only a few per cent.

Since R_Q represents the stabilizing effect of latent heat release at the interface, it is understandable that as the parameter R_Q increases the critical Rayleigh number for a fixed value of S also increases. As can be seen from Figure 1, for sufficiently large R_Q the critical Rayleigh number becomes insensitive to the value of S. Also, for sufficiently large R_Q the critical Rayleigh number for symmetric convection (Figure 1) exceeds the critical Rayleigh number for antisymmetric convection 657.5 [*Chandrasekhar*, 1961]. We find from Figure 1 that $R_{\beta_{crit}}$ decreases as S increases for fixed R_Q. This reflects the fact that as S increases the fractional density change associated with the phase transition becomes increasingly significant as compared with the density change associated with thermal

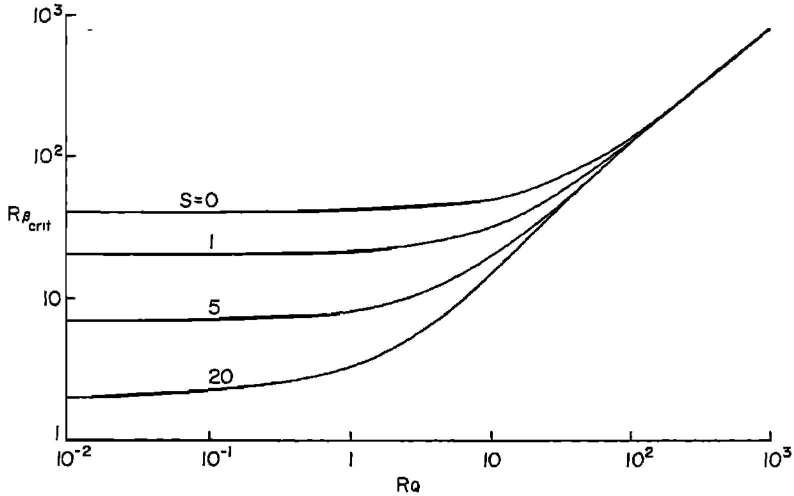

Fig. 1. The minimum critical Rayleigh number $R_{\beta_{crit}}$ for convection through the phase boundary as a function of R_Q for various values of S.

expansion, and the phase change plays a more important role in driving the instability, thus reducing the critical Rayleigh number. For a wide range of values of R_Q and S, the critical Rayleigh number for symmetric convection through a phase change is lower than the critical Rayleigh number for a single-phase fluid.

STABILITY OF THE MANTLE

The results of the above calculations will now be applied to the earth's mantle. A stability calculation will be carried out with a phase change present, and the results will be compared to the calculation without a phase change.

The olivine-spinel phase change will be approximated by a univariant phase-change boundary at an undisturbed depth of 400 km; we therefore take $d = 400$ km. Following *Ringwood* [1970] and others, the mantle above the phase boundary is taken to be 75% olivine with a Mg_2SiO_4 to Fe_2SiO_4 ratio of 9 to 1. There is some uncertainty about the properties of the olivine-spinel phase change. To specify this phase change it is sufficient to give the change in specific volume Δv and the slope of the Clapeyron curve γ. For forsterite (Mg_2SiO_4), the calculated values of *Ahrens and Syono* [1967] are $\Delta v = 2.8$ cm³/mole and $\gamma = 50$ b/°K; the extrapolated experimental values of *Akimoto and Fujisawa* [1968] are $\Delta v = 3.3$ cm³/mole and $\gamma = 62$ b/°K, and the extrapolated experimental values of *Ringwood and Major* [1970] are $\Delta v = 4.6$ cm³/mole and $\gamma = 30$ b/°K. On the basis of these values we take $\Delta\rho/\rho = 0.08$ and $\gamma = 40$ b/°K. From equation 1, with $T = 1800$°K at the phase-change boundary, the heat of reaction is $Q = 40$ cal/g. For other mean properties of the upper mantle we take $\alpha = 3 \times 10^{-5}$ °K^{-1}, $g = 10^3$ cm/sec², $\rho = 3.5$ g/cm³, $c_p = 0.3$ cal/g°K, and $\kappa = 10^{-2}$ cm²/sec.

In the analysis given above, the critical Rayleigh number R_β has been given as a function of the parameters S and R_Q. Using the properties given above, the minimum value of the temperature gradient β–β_a that will lead to instability will be determined as a function of the kinematic viscosity. If $\beta\gamma/\rho g \ll 1$, then the parameter $S = 0.76$. For a particular viscosity R_Q is evaluated, and the critical Rayleigh number R_β is determined from Figure 1. This critical Rayleigh number gives the minimum value of the temperature gradient that will lead to convection. If the value of β is such that $\beta\gamma/\rho g = 0$ (1) then the calculation must be iterated.

The critical temperature gradient β–β_a is given as a function of kinematic viscosity in Figure 2. The dashed line is the stability curve for a layer of fluid of thickness $2d$ without a phase change (Rayleigh stability). The solid line to the right of point A is the stability curve for a layer of fluid with a phase change as determined above. To the left of point A the solid

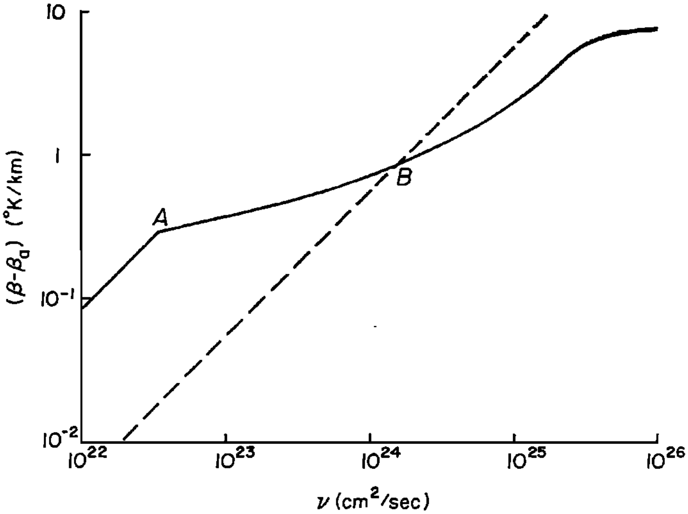

Fig. 2. The minimum critical superadiabatic temperature gradient for instability in the mantle as a function of kinematic viscosity. The dashed line is the stability curve for a mantle without a phase change (Rayleigh stability). The solid line is the stability curve for a mantle with the olivine–spinel phase change. To the left of point A the stability curve for double-cell convection above and below the phase boundary lies below the stability curve for convection through the boundary.

line is the Rayleigh stability curve for a fluid layer of thickness d. In this region the stability curve for double-cell convection lies below the stability curve for convection through the phase boundary.

To the right of point B the phase change is destabilizing (the solid line lies below the dashed line). Between points A and B the phase change is stabilizing but convection takes place through the phase-change boundary. To the left of point A the phase change is absolutely stabilizing and convection does not penetrate the phase boundary. It should be emphasized that these results are based on a linearized stability analysis and are valid only for small amplitude convection. Also the assumption of constant viscosity is probably not a good approximation for the upper mantle. However, the results do show that the olivine–spinel phase change may be destabilizing, partially stabilizing, or an effective barrier to convection. In the absence of convection the geothermal gradient in the upper 800 km of the mantle should be in the range 1°–5°K/km [*Schubert et al.*, 1969]. From Figure 2 we see that this is the range in which the phase change is destabilizing. Therefore the phase change may lead to convection in the deeper mantle where the viscosity is expected to be in the range $\nu = 10^{24} - 10^{26}$ cm^2/sec.

Finite Amplitude Convection

As a further indication of the effect of the olivine–spinel phase change on mantle convection, we consider the phase change in the descending slab of lithosphere beneath oceanic trenches. A typical temperature profile for the mantle [*Turcotte and Oxburgh*, 1969] is given in Figure 3. The Clapeyron curve is included as a dashed line with a constant slope corresponding to $\gamma = 40$ b/°K. The Clapeyron curve intersects the mantle temperature profile at a mean depth of 400 km. Temperature profiles for the descending slab have been calculated by *Oxburgh and Turcotte* [1970]. The minimum temperature in the slab as a function of depth is also included in Figure 3. If the slab is in thermodynamic equilibrium, the phase change will take place in the slab where the Clapeyron curve intersects the temperature profile for the slab. From Figure 3 we see that this occurs at a mean depth of 290 km. The olivine–spinel phase boundary is elevated up to 100 km in the descending slab relative to the rest of the mantle. The additional dense spinel within the slab

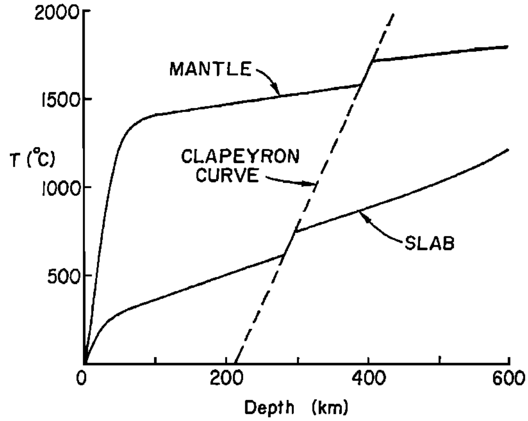

Fig. 3. The mantle temperature profile, the temperature profile for the descending slab of lithosphere at ocean trenches and the Clapeyron curve for the olivine–spinel phase change. The intersections of the temperature profiles with the Clapeyron curve locate the phase change in the mantle and the descending slab.

will exert a body force tending to drive the slab down into the mantle. This is a finite amplitude example of the phase-change instability discussed above.

We next estimate the gravitational body force on the slab due to the elevation of the phase boundary and compare this force with the body force due to the thermal contraction of the slab. For a mean slab width $w = 50$ km, a depth $l = 700$ km, and a mean temperature difference relative to the surrounding mantle $\Delta T = 700$ °K, the gravitational body force on the slab per unit length due to thermal contraction is ($\alpha = 3 \times 10^{-5}$ °K^{-1} and $\rho = 3.5$ g/cm^3)

$$F_\alpha = gwl\rho\alpha\,\Delta T = 2.5 \times 10^{16} \text{ dynes/cm}$$

With an additional depth of spinel $\Delta h = 100$ km and with $\Delta\rho/\rho = 0.08$ for the phase change, the additional gravitational body force per unit length due to the elevation of the phase-change boundary is

$$F_{pc} = gw\Delta h\,\Delta\rho = 1.5 \times 10^{16} \text{ dynes/cm}$$

It is seen that the body force on the descending slab due to the elevation of the phase boundary is nearly as large as the force on the slab due to thermal contraction. Since the sinking of the cold slab into the mantle owing to thermal contraction is a form of finite amplitude Rayleigh convection, this calculation shows that finite amplitude convection due to the phase change instability plays a significant role in mantle convection.

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