One-Dimensional Model of Shallow-Mantle Convection

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One of the models proposed for mantle convection is a counterflow in the asthenosphere to balance the motion of the lithospheric plates. An analysis of this hypothesis has been made, using a model in which the variables depend only on depth. Velocity and temperature profiles are coupled by a temperature- and depth-dependent viscosity. The velocity of the crustal plate and the actual viscosity function are the only inputs to the model. Pressure must increase with distance from the ridge for there to be a return flow at depth and no net mass flow across a vertical section. Horizontal pressure gradients between about 0.1 and 1.0 b/km and shear stresses at the crustal plate between 0.1 and 0.4 kb have been obtained for wide variations in the plate velocity and the viscosity function. However, for these same examples, the surface heat flux is remarkably insensitive to parameter variations; it is between about 0.2 and 0.3 μcal/cm^2 sec. Heating by viscous dissipation provides a self-lubricating mechanism. The higher the plate speed, the lower is the drag on the plate.

It is generally accepted that some type of thermal convection is the driving mechanism for plate tectonics. The lithospheric plates that descend into the mantle at oceanic trenches are cold and dense. The resulting gravitational body force tends to drive the plate down into the mantle. This movement is a form of thermal convection. *Elsasser* [1971] has argued that this body force is entirely responsible for the motion of the surface plates. Since mass must be conserved, a return flow must be associated with the plate motion. *Lliboutry* [1969] and *Jacoby* [1970] argue that this return flow takes place in the asthenosphere at depths between 100 and 300 km. This model is illustrated in Figure 1.

If this model is correct, the flow far from ridges and trenches must be nearly horizontal, and a one-dimensional calculation for the velocity and temperature in the asthenosphere can be made. Heating by viscous dissipation is included since it provides a self-lubricating mechanism; viscous heating increases the temperature, which in turn decreases the viscosity. The heat flux due to viscous dissipation, the horizontal pressure gradient, and the shear stress on the lithosphere will be determined. The pressure gradient will be interpreted in terms of surface elevation and gravitational anomaly. No attempt is made here to consider the mechanism responsible for sea-floor spreading; rather, we assume the velocity of sea-floor spreading to be given and study the resultant motion at depth.

In the analysis diffusion creep is assumed to be the deformation mechanism [*Gordon,* 1965; *Turcotte and Oxburgh,* 1969]. This mechanism leads to a temperature- and depth-dependent Newtonian viscosity, which has a strong minimum in the asthenosphere and is consistent with the above model. An alternative model and deformation mechanism will be discussed in a later section.

MODEL

We take y to be increasing with depth. At $y = 0$ the velocity of spreading u_0 in the $+x$ direction is assumed constant. The velocity $u(y)$ is considered to be horizontal and in the x direction. Thus, the variation of pressure p with depth is hydrostatic; i.e.,

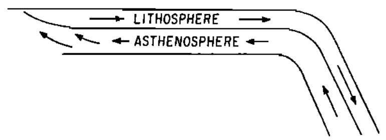

Fig. 1. Model of shallow-mantle convection. Conservation of mass is satisfied by a counterflow in the asthenosphere at depths between 100 and 300 km. The return flow is driven by a hydrostatic pressure that increases with distance from the ridge. This pressure gradient is reflected as an increase in ocean-floor elevation away from the ridge.

$$\partial p/\partial y = \rho g \tag{1}$$

where ρ is the density and g is the acceleration of gravity. A horizontal pressure gradient must balance the viscous retarding force of the shear flow. This balance is expressed by the horizontal momentum equation

$$-\frac{\partial p}{\partial x} + \frac{d}{dy}\left(\mu \frac{du}{dy}\right) = 0 \tag{2}$$

where μ is the dynamic viscosity. The plate drags the flow in the $+x$ direction, while the pressure gradient drives the counterflow in the $-x$ direction. The force on the plate may be a tension due to the body force on the descending part of the plate, or it may be a compression due to gravitational sliding on the ridge flank. In keeping with our one-dimensional approximation, we take the horizontal pressure gradient to be constant. We can thus integrate (2) to obtain

$$\frac{du}{dy} = \frac{1}{\mu}\left(-\tau_0 + \frac{\partial p}{\partial x} y\right) \tag{3}$$

where τ_0 the magnitude of the surface shear stress equals $-\mu\ (du/dy)_{y=0}$. Since the rigid lithospheric plate will tend to drag the material below it, clearly $(du/dy)_{y=0}$ must be negative and τ_0 must be positive.

Integrating (3), we find

$$u = u_0 + \int_0^y \frac{1}{\mu}\left(-\tau_0 + \frac{\partial p}{\partial x} y'\right) dy' \tag{4}$$

which, if the motion is to vanish far from the surface, leads to the constraint

$$0 = u_0 + \int_0^\infty \frac{1}{\mu}\left(-\tau_0 + \frac{\partial p}{\partial x} y'\right) dy' \tag{5}$$

Conservation of mass requires that the net mass flow across a vertical section be zero. Thus

$$0 = \int_0^\infty u\, dy = \int_0^\infty dy \cdot \left[u_0 + \int_0^y \frac{1}{\mu}\left(-\tau_0 + \frac{\partial p}{\partial x} y'\right) dy'\right] \tag{6}$$

The additional constraint imposed by (6) allows us to conclude that $\partial p/\partial x$ must be positive; i.e., pressure must increase with distance from the ridge. This conclusion follows from the fact that, if $\partial p/\partial x$ were negative, u would be everywhere positive (note equation 4) and (6) could not be satisfied.

Since viscosity is a function of temperature T and pressure, μ is in turn a function of depth, and the velocity distribution $u(y)$ cannot be found from (4) without first determining $T(y)$. The temperature distribution can be ascertained from the one-dimensional approximation to the energy equation, which expresses the balance between thermal conduction and viscous dissipation as

$$0 = \frac{d}{dy}\left(k \frac{dT}{dy}\right) + \mu\left(\frac{du}{dy}\right)^2 = \frac{d}{dy}\left(\frac{k\, dT}{dy}\right) + \left(-\tau_0 + \frac{\partial p}{\partial x} y\right)\frac{du}{dy} \tag{7}$$

where k is the thermal conductivity. Since the layer of flow is thin, it is appropriate to neglect heat production by radioactive elements within the layer. We can relate surface shear stress, surface heat flux q_0 (since we are interested in the heat flux out of the upper surface and the direction of positive y points into the surface, we define q_0 as $+k\ (dT/dy_{y=0})$, and velocity of spreading by integrating (7):

$$0 = k \frac{dT}{dy} - q_0 - \tau_0(u - u_0) + \frac{\partial p}{\partial x}\left(yu - \int_0^y u\, dy'\right) \tag{8}$$

In addition to requiring that $u \to 0$ as $y \to \infty$, we also assume that $yu \to 0$ and $k\ dT/dy \to 0$ as $y \to \infty$. Thus, by letting $y \to \infty$ in (8), we find

$$q_0 = u_0 \tau_0 \tag{9}$$

The heat flux generated by viscous dissipation is equal to the product of the velocity of the surface plate and the shear stress on the plate.

METHOD OF SOLUTION

The determination of the temperature and velocity profiles is facilitated by the introduction of the integral of the mass flow through a vertical section U, where

$$U = \int_0^y dy' \left(\int_0^{y'} u \, dy'' \right) \qquad (10)$$

Thus, the velocity is d^2U/dy^2 and (3) becomes

$$\frac{d^3 U}{dy^3} = \frac{-\tau_0 + (\partial p/\partial x)y}{\mu} \qquad (11)$$

We assume that the thermal conductivity is constant (constant thermal conductivity has been shown to be a good approximation to the upper mantle [*Fukao et al.*, 1968]) and integrate (8) to obtain

$$T = T_0 - \frac{1}{k}\frac{dU}{dy}\left(-\tau_0 + \frac{\partial p}{\partial x} y\right) + \frac{2}{k}\frac{\partial p}{\partial x} U \qquad (12)$$

where T_0 is the temperature at $y = 0$.

Given the viscosity function $\mu(T, p)$, we can obtain a single ordinary differential equation for U from (1), (11), and (12). We consider here the class of viscosity functions for which diffusion creep is the deformation mechanism [*Herring*, 1950]:

$$\mu = \frac{KTR_c^2}{10 D_0 V_a} \exp\left(\frac{E^* + pV^*}{KT}\right) \qquad (13)$$

where K is Boltzmann's constant, R_c is the radius of the crystals, D_0 is a reference diffusion coefficient, V_a is the atomic volume, E^* is an activation energy, and V^* is an activation volume.

With the aid of (1), we can write (13) in the form

$$\mu = aT \exp\left[(c + by)/T\right] \qquad (14)$$

where

$$a = \frac{KR_c^2}{10 D_0 V_a} \qquad b = \frac{\rho g V^*}{K}$$
$$c = \frac{E^* + p_0 V^*}{K} \qquad (15)$$

and p_0 is the pressure at $y = 0$. Combining (11), (12), and (14), we obtain

$$\frac{d^3U}{dy^3} = \left\{\left(-\tau_0 + \frac{\partial p}{\partial x} y\right) \cdot \exp\left[\frac{-c - by}{T_0 - \frac{1}{k}\frac{dU}{dy}\left(-\tau_0 + \frac{\partial p}{\partial x} y\right) + \frac{2}{k}\frac{\partial p}{\partial x} U}\right]\right\} \cdot \left\{a\left[T_0 - \frac{1}{k}\frac{dU}{dy}\left(-\tau_0 + \frac{\partial p}{\partial x} y\right) + \frac{2}{k}\frac{\partial p}{\partial x} U\right]\right\}^{-1} \qquad (16)$$

with the boundary conditions

$$U = \frac{dU}{dy} = 0 \qquad \frac{d^2U}{dy^2} = u_0 \qquad \text{at } y = 0 \quad (17)$$

and

$$\frac{dU}{dy} \quad \frac{d^2U}{dy^2} \to 0 \qquad \text{as } y \to \infty \qquad (18)$$

The initial conditions (17) are sufficient to integrate the third-order differential equation (16). The unknowns in the problem, τ_0 and $\partial p/\partial x$, are determined from the required behavior of U as $y \to \infty$ (conditions 18).

The solution of this nonlinear problem must be determined numerically; thus it is difficult to impose conditions 18 at $y = \infty$. Instead we require dU/dy and d^2U/dy^2 to be zero at some finite depth $y = L$. The sensitivity of the solution to the choice of L can be assessed a posteriori. It is convenient to introduce the dimensionless quantities

$$\eta = y/L \quad V = U/u_0L^2 \quad \langle\tau_0\rangle = \tau_0 u_0 L/kT_0$$
$$\left\langle\frac{\partial p}{\partial x}\right\rangle = \frac{\partial p}{\partial x}\frac{u_0 L^2}{kT_0} \qquad \langle\mu_0\rangle = \frac{a u_0^2}{k} e^{c/T_0} \qquad (19)$$

The dimensionless form of the problem becomes

$$\frac{d^3V}{d\eta^3} = \left\{\left(-\langle\tau_0\rangle + \left\langle\frac{\partial p}{\partial x}\right\rangle \eta\right)\left[\exp\frac{c}{T_0} - \frac{(c/T_0)[1 + (bL/c)\eta]}{\left[1 - \left(-\langle\tau_0\rangle + \left\langle\frac{\partial p}{\partial x}\right\rangle \eta\right)\frac{dV}{d\eta} + 2\left\langle\frac{\partial p}{\partial x}\right\rangle V\right]}\right]\right\} \cdot \left[\langle\mu_0\rangle\left(1 - \frac{dV}{d\eta}\left(-\langle\tau_0\rangle + \left\langle\frac{\partial p}{\partial x}\right\rangle \eta\right) + 2\left\langle\frac{\partial p}{\partial x}\right\rangle V\right)\right]^{-1} \qquad (20)$$

$$V = \frac{dV}{d\eta} = 0 \qquad \frac{d^2V}{d\eta^2} = 1 \quad \text{at } \eta = 0 \qquad (21)$$

$$\frac{dV}{d\eta} = \frac{d^2V}{d\eta^2} = 0 \qquad \text{at } \eta = 1 \qquad (22)$$

The input parameters $\langle \mu_0 \rangle$, c/T_0, and bL/c depend on the choice of u_0, L, and the parameters in the viscosity function. The dimensionless shear stress $\langle \tau_0 \rangle$ and pressure gradient $\langle \partial p/\partial x \rangle$ are determined in an iterative manner by integrating (20) from $\eta = 0$ and using a Newton-Raphson procedure for satisfying conditions 22 at $\eta = 1$.

Discussion of Results

Aside from the choice of the velocity of sea-floor spreading u_0 and the depth to which our numerical integration extends L, we must specify the parameters of the viscosity function. *Gordon* [1965] has studied the values of these parameters appropriate to the mantle. His values and estimates of error are $E^* = 5.5$ ev (3–6), $V^* = V_a = 10$ A^3 (5–20), $D_0 = 5$ cm^2/sec (1–10), and $R_c = 0.05$ cm. *Magnitsky* [1967] has given values of the activation energy for several minerals: $E^* = 5.6$ ev for olivine, 5.2 ev for enstatite, and 6.1 ev for diopside. Magnitsky takes D_0 as equal to 300 cm^2/sec. One of the most uncertain parameters is the crystalline radius R_c. *Orowan* [1967] has assumed that individual mantle crystals may be very large (even kilometer scale), whereas the crystal sizes in xenoliths believed to be of mantle origin are no larger than about 1 cm. Because of the exponential dependence of viscosity on the reciprocal temperature, the uncertainties in these parameters lead to viscosities that may differ by many orders of magnitude at depth. Thus, in the following, we will explore the results of wide variations in the viscosity function parameters.

To illustrate the numerical results obtained, we show in Figure 2 the dependence of the dimensionless velocity $u/u_0 = d^2V/d\eta^2$, the dimensionless temperature

$$T/T_0 - 1 = -\left(-\langle \tau_0 \rangle + \left\langle \frac{\partial p}{\partial x} \right\rangle \eta\right) \frac{dV}{d\eta} + 2V \left\langle \frac{\partial p}{\partial x} \right\rangle$$

and the dimensionless viscosity μ/μ_0 (μ_0 is the viscosity at $y = 0$) on η for $L = 600$ km, $u_0 = 1$ cm/year, $\rho = 3.3$ g/cm^3, $k = 7 \times 10^{-3}$ cal/cm sec °K, $E^* = 4.5$ ev, $V_a = 10$ A^3, $V^* = 25$ A^3, $D_0 = 20$ cm^2/sec, and $R_c = 0.2$ cm. Aside from the value of V^*, which they took to be 15 A^3, *Turcotte and Oxburgh* [1969] chose these values of the crystalline properties in their investigation of the effects of variable physical properties on mantle convection. For the case shown in Figure 2 the viscosity decreases rapidly with depth and attains a minimum value 5 orders smaller than μ_0 ($\mu_0 = 2.6 \times 10^{27}$ poise) at a depth of 270 km. The velocity remains substantially at its surface value down to depths of about 150 km as a result of the large viscosities in this region. The velocity falls rapidly to zero as the viscosity decreases with further increase in depth. The region of reverse flow is seen to coincide approximately with the depth range, where the viscosity values are minimum. Values of the surface shear stress and horizontal pressure gradient for the parameters of Figure 2 are 0.35 kb and 1.1 b/km, respectively. The corresponding surface heat flux is 0.26 μcal/cm^2 sec. At depths less than a few tens of kilometers, the viscosity is so large that it is essentially infinite for computational purposes. Thus, in the calculations leading to Figure 2 and in the other examples to be discussed, $y = 0$ has been assumed to correspond to the depth where the temperature is sufficiently high for the viscosity to be not excessively large. The temperature T_0 has been

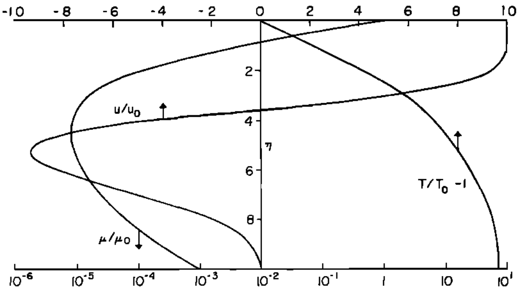

Fig. 2. The dimensionless velocity u/u_0, the dimensionless temperature $(T/T_0) - 1$, and the dimensionless viscosity μ/μ_0 as a function of depth. The parameter values are $L = 600$ km, $u_0 = 1$ cm/year, $\rho = 3.3$ g/cm^3, $k = 7 \times 10^{-3}$ cal/cm sec °K, $E^* = 4.5$ ev, $V_a = 10$ A^3, $V^* = 25$ A^3, $D_0 = 20$ cm^2/sec, and $R_c = 0.2$ cm. The counterflow occurs predominantly in the depth range where the viscosity is minimum.

taken to be 846°C [*Turcotte and Oxburgh,* 1969].

Results of other numerical calculations are summarized in Table 1. Values of any parameters not explicitly given in Table 1 are those used in the computations of Figure 1. Since E^* and V^* appear in the exponential dependence of the viscosity, the values chosen represent a change of several orders of magnitude in the viscosity. The activation energy E^* affects the temperature dependence, and the activation volume V^* affects the depth dependence. The heat flux due to viscous dissipation, the pressure gradient, and the shear stress on the lithosphere are shown for each case. Over the wide range of conditions considered, the heat flux generated by viscous dissipation remains almost constant.

TABLE 1. Summary of Computational Results

Parameter	τ_0, kb	$\partial p/\partial x$, b/km	q_0, μcal/cm^2 sec
	$E^* = 4.5$ ev		
$V^* = 15\ A^3$	0.24	0.60	0.18
$V^* = 20\ A^3$	0.29	0.80	0.22
$V^* = 25\ A^3$	0.35	1.10	0.26
	$V^* = 25\ A^3$		
$E^* = 4.5$ ev	0.35	1.10	0.260
$E^* = 4.75$ ev	0.36	1.0	0.265
$E^* = 5.0$ ev	0.37	0.94	0.270
	$V^* = 15\ A^3$, $E^* = 4.5$ ev		
$u_0 = 1$ cm/year	0.24	0.60	0.18
$u_0 = 2$ cm/year	0.13	0.31	0.19
$u_0 = 4$ cm/year	0.07	0.16	0.21

Heat Flow

Our numerical calculations indicate that the flow is self-adjusting; thus the heat flux generated by viscous dissipation q_0 is nearly a constant with a value of 0.24 (±20%) μcal/cm^2 sec for the range of parameters investigated here. This heat flux is into the lithosphere, which is actually the thermal boundary layer that develops in the surface plate as it moves away from the oceanic ridge [*Turcotte and Oxburgh,* 1969]. It is appropriate to add q_0 to the heat flux from the boundary layer to determine the heat flux to the ocean floor. This step could help to explain the discrepancy between the predictions of the boundary-layer theory and the measurements that show a nearly uniform surface heat flux of 1 μcal/cm^2 sec away from oceanic ridges.

Shear Stress

The frictional heating being nearly constant, it follows from (9) that the shear stress on the surface plate is inversely proportional to the velocity of the plate and is given by

$$\tau_0 \approx 10/u_0 \qquad (23)$$

with u_0 in centimeters per second and τ_0 in dynes per square centimeter. The faster the lithosphere moves, the less the resistance to the motion. Viscous dissipation provides a self-lubricating mechanism, which is more effective for higher velocities.

These values of shear stress can be compared with those obtained by *Lliboutry* [1969], using an approximate variational method. For the western Pacific, Lliboutry takes $u_0 = 7.2$ cm/year and obtains $\tau_0 = 8.6$ bars; from (23), we find $\tau_0 \approx 44$ bars. For South America, Lliboutry takes $u_0 = 0.4$ cm/year and obtains $\tau_0 = 0.48$ bar; from (23), we find $\tau_0 \approx 790$ bars. Lliboutry used constant viscosity and found that the shear stress was proportional to the velocity. According to the results given here, this procedure leads to serious errors.

Pressure Gradient

A positive pressure gradient is required to provide the counterflow at depth. This pressure gradient will be reflected as an increase in elevation of the ocean floor in the direction of flow. The hydrostatic head associated with the elevated topography provides the pressure gradient. This process is in essence negative gravitational sliding.

The pressure gradient is related to the increase in elevation by

$$dp/dx = \rho g\, dh/dx \qquad (24)$$

If $dp/dx = 10^{-1}$ b/km, then $dh/dx = 5 \times 10^{-4}$ or ½ km per 1000 km. Across the Pacific, this increase in elevation would amount to 5 km; the Pacific adjacent to the East Pacific rise would be 5 km deeper than the Pacific at the Japanese trench. However, there is no evidence that this increase in elevation occurs; in fact, the evidence is that the Pacific deepens away from the ridge and toward the trench.

The positive pressure gradient would also cause a positive gravity anomaly as one moves away from a ridge flank. For an elevation of topography of 4 km the gravity anomaly would be 2000 mgal. This value is more than an order of magnitude larger than observed gravitational anomalies.

CONCLUSIONS

The one-dimensional calculations predict that elevation should increase as one moves away from a ridge toward a trench and that a large positive gravitational anomaly should appear. Since neither condition is observed, we conclude that the counterflow is not restricted to the asthenosphere. To explain the observation, it is necessary that the return flow take place at considerably greater depth, possibly below 700 km, and that the mantle beneath the lithosphere convect in the same direction as the movement of the lithosphere. This model is illustrated in Figure 3.

Runcorn [1962*a*, *b*], *Girdler* [1963], and others have favored convection throughout the entire mantle. This view was questioned on the basis of the calculations of *Munk and MacDonald* [1960], who required a viscosity of 10^{25} poises for the deep mantle to explain the earth's excess equatorial bulge as a fossil bulge left over from the time when the earth was rotating more rapidly. However, *Dicke* [1969] has considered the relaxation time for a second-order harmonic distortion of the earth and has concluded that the viscosity of the lower mantle is near 10^{22} poises. *Goldreich and Toomre* [1969] have questioned Munk and MacDonald's explanation of the excess equatorial bulge and have argued that the viscosity of the lower mantle is in the range 10^{22}–10^{24} poises to explain polar wandering. Also, *Knopoff* [1964] argued that convection would not penetrate the phase changes that are known to take place in the mantle near depths of 400, 700, and possibly 1000 km. However, *Schubert et al.* [1970] and *Schubert and Turcotte* [1971] have shown that at least the 400-km phase change is expected to enhance convection.

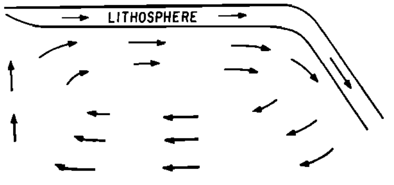

Fig. 3. Model of deep-mantle convection. The return flow necessary to establish conservation of mass takes place throughout the upper mantle and possibly throughout the entire mantle.

Gordon [1965] and *Turcotte and Oxburgh* [1969] have shown that with diffusion creep as the deformation mechanism one can expect the viscosity of the mantle to increase rapidly with depth. The decrease in the diffusion coefficient with increasing pressure dominates over the increase with temperature. However, *Weertman* [1970] has argued that dislocation motion is the dominant deformation mechanism in the mantle and shown that the increase in the equivalent viscosity with depth is much less with this deformation mechanism. The critical parameter in determining which deformation mechanism dominates is the mean crystal size in the mantle. If the mean crystal size is 0.1 cm, dislocation glide will dominate at stresses greater than about 10 bars; if the mean crystal size is 1 cm, dislocation glide will dominate at stresses greater than 1 bar.

We conclude that the hypothesis of a return flow in the asthenosphere is not valid and that significant mantle convection occurs throughout the upper mantle and possibly throughout the entire mantle. It is likely that the dominant deformation mechanism is dislocation glide.

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